1. Expand each of the following functions in a Fourier sine series of period $(2l)$, over the interval $(0, l)$, and in each case sketch the function represented by the series in the interval $(-3l, 3l)$:

(a) $f(x) = \begin{cases} 0 & (x < 0) \\ x(l-x) & (x > 0), \end{cases}$

(b) $f(x) = \begin{cases} 0(x < 0) \\ x & (0 < x < \frac{l}{2}) \\ l-x & (x > \frac{l}{2}) \end{cases}$

(c) $f(x) = \sin \frac{\pi x}{2l}$

(d) $f(x) = \begin{cases} \frac{1}{\varepsilon} & (0 < x < \varepsilon < l) \\ 0 & Otherwise \end{cases}$

2. (a) Obtain the expansion

$$\cos \alpha x = \frac{\sin \pi \alpha}{\pi \alpha} + \sum_{n=1}^{\infty} (-l)^n \frac{2\alpha \sin \pi \alpha}{\pi (\alpha^2 - n^2)} \cos(nx)$$
\((-\pi \leq x \leq \pi)\) when \(a\) is nonintegral.

(b) Deduce from this result that \(\cot \pi a = \frac{1}{\pi} \left( -\sum_{n=1}^{\infty} \frac{2a}{n^2 - a^2} \right)\), when \(a\) is a nonintegral.

3. Expand each of the functions listed in Problem 1, (a through d, above), in a Fourier series of period 2l, over the interval \((-l, l)\), and in each case sketch the function represented by the series in the interval \((-3l, 3l)\).

4. (a) If the representation \(f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{l} (0 \leq x \leq l)\) is valid, show formally that \(\frac{2}{l} \int_{0}^{l} [f(x)]^2 \, dx = 2A_0^2 + \sum_{n=1}^{\infty} A_n^2\).

(b) If the representation \(f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} (0 < x < l)\) is valid, show formally that \(\frac{2}{l} \int_{0}^{l} [f(x)]^2 \, dx = \sum_{n=1}^{\infty} B_n^2\).