In[8]:=
```
P[a_, b_] := (1 + a x + I b x) / (1 + a x - I b x)
```

In[206]:=
```
Clear[a4, a1, a2, a3, b]
```

In[323]:=
```
prop = P[a, dz b1] P[a , dz b2]
```

Out[323]=
$$\frac{(1 + a\ x + i\ b1\ dz\ x)\ (1 + a\ x + i\ b2\ dz\ x)}{(1 + a\ x - i\ b1\ dz\ x)\ (1 + a\ x - i\ b2\ dz\ x)}$$

In[243]:=
```
target = Exp[ I dz (Sqrt[1 + x] - 1)]
```

Out[243]=
$$e^{i\ dz\ \left(-1+\sqrt{1+x}\right)}$$

In[324]:=
```
constr = Collect[Simplify[Normal[Series[target - prop, {x, 0, 4}]]], x]
```

Out[324]=
$$-\frac{1}{2} i\ (-1 + 4\ b1 + 4\ b2)\ dz\ x + \frac{1}{8} dz\ \left(-i + 16\ i\ a\ (b1 + b2) + \left(-1 + 16\ b1^2 + 32\ b1\ b2 + 16\ b2^2\right) dz\right) x^2 + \frac{1}{48} i\ dz\ \left(3 - 96\ a^2\ (b1 + b2) - 3\ i\ dz + 192\ i\ a\ (b1 + b2)^2\ dz + \left(-1 + 96\ b1^3 + 192\ b1^2\ b2 + 192\ b1\ b2^2 + 96\ b2^3\right) dz^2\right) x^3 + \frac{1}{384} dz\ \left(-15\ i + 768\ i\ a^3\ (b1 + b2) - 15\ dz + 2304\ a^2\ (b1 + b2)^2\ dz + 6\ i\ dz^2 - 2304\ i\ a\ \left(b1^3 + 2\ b1^2\ b2 + 2\ b1\ b2^2 + b2^3\right) dz^2 - \left(-1 + 768\ b1^4 + 1536\ b1^3\ b2 + 1536\ b1^2\ b2^2 + 1536\ b1\ b2^3 + 768\ b2^4\right) dz^3\right) x^4$$

In[325]:=
```
eqns = Table[ComplexExpand[Coefficient[constr, x^l]], {l, 1, 4}]
```

Out[325]=
$$\left\{i\ \left(\frac{dz}{2} - 2\ b1\ dz - 2\ b2\ dz\right), -\frac{dz^2}{8} + 2\ b1^2\ dz^2 + 4\ b1\ b2\ dz^2 + 2\ b2^2\ dz^2 + i\ \left(-\frac{dz}{8} + 2\ a\ b1\ dz + 2\ a\ b2\ dz\right), \frac{dz^2}{16} - 4\ a\ b1^2\ dz^2 - 8\ a\ b1\ b2\ dz^2 - 4\ a\ b2^2\ dz^2 + i\ \left(\frac{dz}{16} - 2\ a^2\ b1\ dz - 2\ a^2\ b2\ dz - \frac{dz^3}{48} + 2\ b1^3\ dz^3 + 4\ b1^2\ b2\ dz^3 + 4\ b1\ b2^2\ dz^3 + 2\ b2^3\ dz^3\right), -\frac{5\ dz^2}{128} + 6\ a^2\ b1^2\ dz^2 + 12\ a^2\ b1\ b2\ dz^2 + 6\ a^2\ b2^2\ dz^2 + \frac{dz^4}{384} - 2\ b1^4\ dz^4 - 4\ b1^3\ b2\ dz^4 - 4\ b1^2\ b2^2\ dz^4 - 4\ b1\ b2^3\ dz^4 - 2\ b2^4\ dz^4 + i\ \left(-\frac{5\ dz}{128} + 2\ a^3\ b1\ dz + 2\ a^3\ b2\ dz + \frac{dz^3}{64} - 6\ a\ b1^3\ dz^3 - 12\ a\ b1^2\ b2\ dz^3 - 12\ a\ b1\ b2^2\ dz^3 - 6\ a\ b2^3\ dz^3\right)\right\}$$

In[326]:=
```
solb1 = Simplify[Solve[{eqns[[1]] == 0}, {b1}][[1]]]
```

Out[326]=
$$\left\{b1 \to \frac{1}{4} - b2\right\}$$

In[327]:=
```
Simplify[ComplexExpand[Simplify[eqns[[2]]] /. solb1]]
```

Out[327]=
$$\frac{1}{8} i\ (-1 + 4\ a)\ dz$$

In[328]:=
```
a = 1 / 4
```

Out[328]=
$$\frac{1}{4}$$

In[330]:=
```
eqns2 = Simplify[eqns /. solb1]
```

Out[330]=
$$\left\{0,\ 0,\ \frac{1}{96}\ i\ dz\ \left(3+\left(1-12\ b2+48\ b2^2\right)\ dz^2\right),\right.$$
$$\left.-\frac{1}{384}\ dz\ \left(12\ i+6\ dz+3\ i\ \left(1-12\ b2+48\ b2^2\right)\ dz^2+\left(2-24\ b2+96\ b2^2\right)\ dz^3\right)\right\}$$

In[332]:=
```
solb2 = Solve[eqns2[[3]] == 0 , b2]
```

Out[332]=
$$\left\{\left\{b2\to\frac{3\ dz^2-\sqrt{3}\ \sqrt{-12\ dz^2-dz^4}}{24\ dz^2}\right\},\ \left\{b2\to\frac{3\ dz^2+\sqrt{3}\ \sqrt{-12\ dz^2-dz^4}}{24\ dz^2}\right\}\right\}$$

No real solution !