0.0.1 Calculation of the longitudinal vector component

Summary:

- Vectorial nature of light can be easily implemented into spectral beam propagation methods.
- The longitudinal component of the electric field amplitude need not be simulated along the whole propagation distance. Rather, it can be calculated from the divergence condition only when needed.

Background:
We pointed out in the introductory section that the beam propagation method usually works only with the transverse field components, whether electric or magnetic. But sometimes, one needs the longitudinal vector component of the field, for example for calculation of a nonlinear response.

It is possible to write the evolution equations for the longitudinal field component, and these can be solved along with the equations for the dominant polarization(s). This becomes especially simple in the case of a spectral method applied to a homogeneous medium, since the algorithm is exactly the same as for the transverse field components. Different polarization components are completely de-coupled, and each satisfies the same wave equation. The identical FFT-BPM algorithm therefore operates on each polarization component of the electric field. Naturally, one must make sure that the initial condition exhibits zero divergence, otherwise the solution could not represent a solution to Maxwell equations. However, such an approach would be unnecessary...

In the case of spectral FFT-based method, the longitudinal component can be obtained easily without actually "propagating" it. This is possible because the simulated medium is homogeneous, and the divergence constraint can yield the z-component exactly. The electric Gauss law in the real-space representation,

$$\nabla.\vec{E}(x,y,z) = 0$$

translates into the spectral-space representation

$$A_x(k_x,k_y,z)k_x + A_y(k_x,k_y,z)k_y + k_z A_z(k_x,k_y,z) = 0$$

where the z component of the wavevector can be fixed by requiring that the dispersion relation is satisfied, namely

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2 n(\omega)^2}{c^2} \ .$$

Thus, having calculated $A_{x,y}(k_x,k_y,z)$ for the given z where E_z is required, one transforms the $E_{x,y}(x,y,z)$ into spectral representation, and a calculates

$$A_z(k_x,k_y,z) = -\frac{A_x k_x + A_y k_y}{\sqrt{k_0^2 - k_x^2 - k_y^2}} \ .$$

This formula must of course be restricted to those wave-vectors that result in a real-valued k_z. Testing for this is usually necessary because the longitudinal components only become significant in non-paraxial regimes. In such situations, BPM usually utilizes spectral space grid (k_x,k_y) that encompasses regions corresponding to evanescent waves. They would normally carry negligible amplitudes.

Task 1:

- Starting from the implementation of the 2D FFT-BPM from the previous practice package, add a function which will calculate the E_z component of the electric field. You may assume that $E_y = 0$, or that the dominant field components is oriented along axis x.
- Set up a simulation to illustrate that a significant longitudinal field evolves in the focal point of a beam. Note the ratio between the field strength E_X and E_z, and how it depends on the focal length.

Solution:
Instructor's solution example is stored in *pEzInFocus.m*, for a simulation showing a tightly focused beam. It executes the "scalar" (single polarization component) simulation exactly as before. When the final propagation distance is reached, the E_z field is calculated, using the formula above, as follows:

Listing 1: Calculation of the longitudinal field

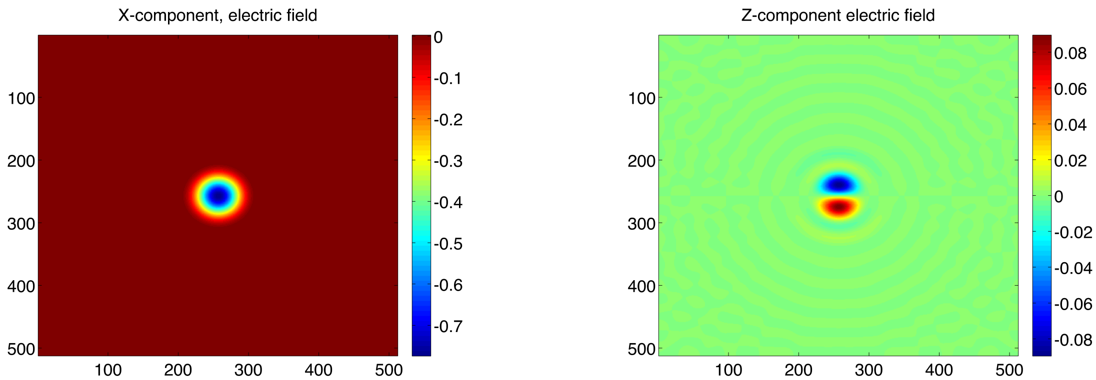
```
% calculate the spatial spectrum of Ex:
spatialspectrum = fft2(amplitudeX);

% evaluate the wave-number dependent factor Ez/Ex:
operator = zeros(NX,NX);
for x=1:NX
    for y=1:NX
        if( k0^2 > kx(x)^2 + kx(y)^2 )
         operator(x,y) = -kx(x)/sqrt(k0^2 - kx(x)^2 - kx(y)^2);
        end
    end
end

% apply to spatial spectrum and transform back to real space
amplitudeZ = ifft2(spatialspectrum.*operator);
```

To emphasize the strength of the longitudinal vector component, one needs a tight focus. One convenient was to set up a simulation is to use the analytic Gaussian beam formula evaluated at a given distance before the beam focus. In the example that follows the beam waist was chosen equal to the wavelength $\lambda = 633$nm. Note that the fact that this analytic formula does not describe accurately tightly focused beam solutions is irrelevant: It does represent *some* initial condition, and the longitudinal part is not initially specified. The beam is propagated into focus, and E_Z is calculated there. The result is illustrated in the figure that follows.

Comparison of the two panels shows that the intensity ratio E_z/E_x is about one tenth. Thus, even in the most tightly focused beam, the longitudinal component is relatively weak. This is one of the rationales for neglecting E_z in many beam propagation approaches.



Dominant E_x (left) and longitudinal E_z (right) fields in the focal point. Propagation of E_x was simulated over four Rayleigh range length, and then E_z was evaluated form the $\nabla\dot{\mathbf{E}} = 0$ condition. Note the ratio between the field strengths in the two panels. Horizontal and vertical axes correspond to array indices of the computational grid.

Task 2:
Set up a similar simulation illustrating the longitudinal field component in a beam that propagates at a steep angle w.r.t. the axis. The high-angle Gaussian beam solution developed previously can be utilized to create an initial condition.

The instructor‘s example can be inspected in *pEzAtSteepAngle.m*.