

In[2]:= **eqn1** = -F[0, 1] + F[1, 0] + r (F[1, 1] - F[0, 0])

Out[2]:= -F[0, 1] + F[1, 0] + r (-F[0, 0] + F[1, 1])

In[5]:= **Ansatz[x_, t_] := 1 Exp[-I k x - I om t] + R Exp[+I k x - I om t]**

In[6]:= **eqn2** = **eqn1** /. F[a_, b_] -> Ansatz[a, b]

Out[6]:= $e^{-i k x - i \omega t} + e^{i k x - i \omega t} R + r (-1 + e^{-i k x - i \omega t} - R + e^{i k x - i \omega t} R)$

In[9]:= **solR** = **Simplify[Solve[eqn2 == 0, R][[1]]]**

Out[9]:= $\left\{ R \rightarrow \frac{e^{-i k x} (e^{i k x} - e^{i \omega t} - r + e^{i (k x + \omega t)} r)}{-1 + e^{i (k x + \omega t)} + e^{i k x} r - e^{i \omega t} r} \right\}$

In[15]:= **aux1** = **Simplify[ExpToTrig[R /. solR]]**

Out[15]:=
$$\frac{(\cos[k] - i \sin[k]) \left(\sin\left[\frac{k - \omega}{2}\right] + r \sin\left[\frac{k + \omega}{2}\right] \right)}{r \sin\left[\frac{k - \omega}{2}\right] + \sin\left[\frac{k + \omega}{2}\right]}$$

In[16]:= **aux2** = **TrigExpand[aux1]**

Out[16]:=
$$\begin{aligned} & - \left(i \cos\left[\frac{k}{2}\right] \cos\left[\frac{\omega}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) - \\ & \left(i r \cos\left[\frac{k}{2}\right] \cos\left[\frac{\omega}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) + \\ & \left(i \cos\left[\frac{k}{2}\right]^3 \cos\left[\frac{\omega}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) + \\ & \left(i r \cos\left[\frac{k}{2}\right]^3 \cos\left[\frac{\omega}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) - \\ & \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) - \\ & \left(r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) + \\ & \left(3 \cos\left[\frac{k}{2}\right]^2 \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) + \\ & \left(3 r \cos\left[\frac{k}{2}\right]^2 \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] \right) / \\ & \left(2 \left(\cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + r \cos\left[\frac{\omega}{2}\right] \sin\left[\frac{k}{2}\right] + \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] - r \cos\left[\frac{k}{2}\right] \sin\left[\frac{\omega}{2}\right] \right) \right) - \end{aligned}$$

[illegible]

$$\begin{aligned}
& \left(2 \left(\cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + r \cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] - r \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] \right) \right) - \\
& \left(3 r \cos \left[\frac{k}{2} \right] \sin \left[\frac{k}{2} \right]^2 \sin \left[\frac{\omega m}{2} \right] \right) / \\
& \left(2 \left(\cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + r \cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] - r \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] \right) \right) - \\
& \left(i \sin \left[\frac{k}{2} \right]^3 \sin \left[\frac{\omega m}{2} \right] \right) / \\
& \left(2 \left(\cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + r \cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] - r \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] \right) \right) + \\
& \left(i r \sin \left[\frac{k}{2} \right]^3 \sin \left[\frac{\omega m}{2} \right] \right) / \\
& \left(2 \left(\cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + r \cos \left[\frac{\omega m}{2} \right] \sin \left[\frac{k}{2} \right] + \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] - r \cos \left[\frac{k}{2} \right] \sin \left[\frac{\omega m}{2} \right] \right) \right)
\end{aligned}$$

In[19]:= **aux3 = Simplify[(aux2 /. Cos[om/2] → Sqrt[1 - Sin[om/2]^2]) /. Sin[om/2] → dtoverdx Sin[k/2]]**

Out[19]=
$$\frac{\left(2 \operatorname{dtoverdx} (-1 + r) \cos \left[\frac{k}{2} \right] + (1 + r) \sqrt{4 - 2 \operatorname{dtoverdx}^2 + 2 \operatorname{dtoverdx}^2 \cos[k]} \right) (\cos[k] - i \sin[k])}{2 \left(-\operatorname{dtoverdx} (-1 + r) \cos \left[\frac{k}{2} \right] + (1 + r) \sqrt{1 - \operatorname{dtoverdx}^2 \sin \left[\frac{k}{2} \right]^2} \right)}$$

In[25]:= **check0 = Simplify[Simplify[aux3 /. k → 0] /. r → (dtoverdx - 1) / (dtoverdx + 1)]**

Out[25]= 0

In[28]:= **aux4 = Simplify[(aux3 /. k → 2 Pi / n) /. r → (dtoverdx - 1) / (dtoverdx + 1)]**

Out[28]=
$$\frac{\left(-2 \cos \left[\frac{\pi}{n} \right] + \sqrt{4 - 2 \operatorname{dtoverdx}^2 + 2 \operatorname{dtoverdx}^2 \cos \left[\frac{2 \pi}{n} \right]} \right) (\cos \left[\frac{2 \pi}{n} \right] - i \sin \left[\frac{2 \pi}{n} \right])}{2 \left(\cos \left[\frac{\pi}{n} \right] + \sqrt{1 - \operatorname{dtoverdx}^2 \sin \left[\frac{\pi}{n} \right]^2} \right)}$$

In[34]:= **Plot[Abs[aux4 /. dtoverdx → 0.25], {n, 2, 15}, PlotRange → All]**

