

What is OPTI-547 about

Beam Propagation Method (for purposes of this course)

- Family of numerical simulation methods for continuous-wave (CW) regimes
- Mainly utilized to by-pass (expensive) Maxwell simulations
- Why? Certain problems will remain impossible to attack on “first principles” level, no matter what computer power one can use. The first chapter of the course should make this point abundantly clear.
- Beam Propagation Method (BPM) comes in a number of flavors - it is important to understand common principles (even if only used from a black box of commercial package)
- BPM elements (tools) applied in many different fields
- In particular, BPM numerical techniques also useful in ultra-fast NL optics
- BPM still in development, there is an will be no “complete solution”

Course Goals

- Solid grasp of theoretic foundations (physics and numerical mathematics)
- Practical experience in writing simulation tools
- Practical experience in running simulations
- Ability to apply lessons learned in much wider contexts

Course Structure

- Theory track
- Practice track
- Homework practice

What is OPTI-547 about

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About Lecture Notes

Notes are provided as supplementary material for the OPTI-547 course on Beam Propagation Methods in Computational Optics. Section will be posted on the website before the classes.

Selected section will be assigned for reading prior the class.

Additional theoretical material will be included in the Exercise Packages

Additional material will be provided in the course handouts (website)

Exercise Topics(2013 examples)

WP01-1D-Maxwell: Initial conditions, dispersion, stability
WP02-1D-Maxwell: Absorbing boundary implementation
WP03-FFT-BPM: Classical spectral method
WP04-FFT-BPM: Far-Field and limitations
WP05-2D-FFT-BPM: Bright spot of Arago
WP06-DHT-BPM: Hankel-transform based spectral method
WP07-Instability: Basics of finite-difference methods
WP08-FD-CrankNicolson: General matrix implementation
WP09-FD-CN-tridiagonal: Often used building block in numerics
WP10-CN-vs-DHT: Comparative simulations
WP11-FD-CrankNicolson-2D: Two transverse dimensions
WP12-FD-CN: Absorbing boundary conditions
WP13-FD-CN: SpatialSolitons
WP13-FD-CN: SpatialSolitons-HighOrder
WP14-FD-CN: SelfFocusingCollapse
WP15-MOL-basic: Method of lines
WP16-PML: Perfectly matched layer absorbing boundaries
WP17-CN-PML-WG: Application to wave-guide simulation
WP18-ICN: Iterated Crank-Nicolson for fully-vector simulations
WP19-WA-Pade-2D: Pade based wide-angle method
WP20-WA-BPM: Padeization of evolution operators

Course Organization

- Prerequisites:
 - Maxwell equations
 - *some* Matlab experience and/or any other programming language
 - this is a self-contained course: we will refresh all basics needed
- Not-Prerequisites:
 - Numerical mathematics coursework
- Theory and Practical tracks: Every notion covered in the Theory Track will be practiced
- Material covered along a “spiral trajectory:”
 - topics revisited with increasing complexity
 - topics revisited in different contexts
 - small increments + exercises + homework
- Programming
 - Mostly Matlab (because available)
 - Occasionally, other languages used to illustrate techniques
 - Some program writing required: Usually starting from a “template” or from a previous solution
 - Not covered: design and “good practices” for large-scale projects

- Practical sessions:
 - we will implement programs
 - illustrate working and failure modes in mock simulations
 - students are encouraged to join (with their laptops)
 - “workshop” sessions/classes may be organized
 - templates for numerical exercises provided
- Tests:
 - None
 - Homework is a kind of take-home test
- Homework:
 - *develops, extends* material covered in class
 - varying degrees of difficulty
 - often re-visited in following lectures
 - both theory (derivations, literature research), and practical (algorithm implementation, numerical exercises)
 - deliverables in electronic form to instructor
 - other than Matlab languages accepted
 - normally due next week (before 05:00)

- Homework Policy:
 - *Something* must be submitted on the due date
 - Topic discussed in the class after due date
 - * elegant solutions shared with the class
 - * nice bugs and mistakes pointed out (authors remain anonymous)
 - * instructor's solution sometimes shown in part or in full
 - Students can re-submit their solutions arbitrary number of times
 - There is absolutely no penalty for
 - * repeated submissions
 - * asking instructor for help
(and thus revealing in the process an unfinished, buggy, ugly, incorrect , ... solution)
 - * utilizing insight from the post-due-date discussion
- Special Homework(s):
 - there will be one homework for which the above does not apply
 - equals a take-home test
 - this assignment can not be re-submitted
(but it can be discussed with the instructor prior to submission)
 - select sections of notes will be assigned as pre-class reading. Students will be required to turn in their annotated notes for part of the homework credit.

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- Final Projects (big part of your grade):
 - Research oriented
 - Algorithm oriented
 - Software implementation component
 - Simulation component
 - Selected by students from an instructor-given portfolio
 - Customized, individual topics possible
 - More demanding than homework projects — it will require corresponding time!
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- Typical class:
 - Theory session. Derivation plus understanding equations governing a method
 - Implementation. Core parts of the method implementation described
 - Mock simulations. Testing and breaking the method.
 - Sometimes: Discussion of instructor's solution of a homework to be assigned.

Communication

Website:

- http://acms.arizona.edu/FemtoTheory/MK_personal/opti547
- download area for homework, simulation templates, and other material

Office hours:

- time slot TBD
- by appointment
- open-door (call or accept the risk of instructor not being available)

Email:

- kolesik@acms.arizona.edu
- this is where you “hand in” homework
- always include subject: OPTI-547

File formats:

- Please: NO SPACES in filenames, use underscore
- Reports always in pdf
- Zip or tar your hmwk solution into: OPTI547-your_name-some_label (.tar or .zip)

What you need

before next class:

- A computer to run exercises on. Small laptop should be sufficient.
- Matlab
- if so inclined: Mathematica, C, or other “system” to create and run simple programs

optional:

- QtGrace for quick plotting/visualization of results