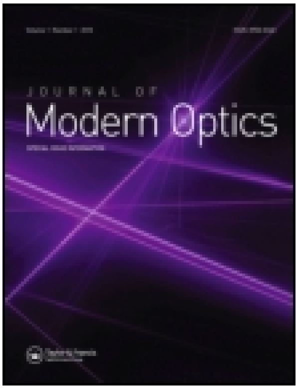


Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop20

Non-diffractive Vector Bessel Beams

Zdeněk Bouchal [a] & Marek Olivík [a]
[a] Department of Optics, Palacký University, 17. listopadu 50, 772 07, Olomouc, Czech Republic

Available online: 01 Mar 2007

To cite this article: Zdeněk Bouchal & Marek Olivík (1995): Non-diffractive Vector Bessel Beams, Journal of Modern Optics, 42:8, 1555-1566

To link to this article: http://dx.doi.org/10.1080/09500349514551361

JOURNAL OF MODERN OPTICS, 1995, VOL. 42, NO. 8, 1555–1566

Non-diffractive vector Bessel beams

ZDENĚK BOUCHAL† and MAREK OLIVÍK
Department of Optics, Palacký University, 17. listopadu 50,
772 07 Olomouc, Czech Republic

(*Received 14 November 1994 and accepted 30 January 1995*)

Abstract. The non-diffractive vector Bessel beams of an arbitrary order are examined as both the solution to the vector Helmholtz wave equation and the superposition of vector components of the angular spectrum. The transverse and longitudinal intensity components of the vector Bessel beams are analysed for the radial, azimuthal, circular and linear polarizations. The radially and azimuthally polarized beams are assumed to be formed by the axicon polarizers used with the initially unpolarized or linearly polarized light. Conditions in which the linearly polarized Bessel beams can be approximated by the scalar solutions to the wave equation are also discussed.

1. Introduction

The concept of the so-called diffractionless propagation of electromagnetic waves, proposed for the first time by Durnin [1, 2], plays an important role in optics as it provides convenient techniques for reducing an inevitable diffractive spreading and enhancing beam directivity. Exact solutions to the scalar wave equation known as Bessel beams [4], Gauss–Bessel beams [5–7] and Weber beams [8] were obtained for free-space propagation. The aperture realizations of such beams were simulated numerically to examine properties of the realistic nearly non-diffracting fields [8–10]. Various apodization techniques were proposed to eliminate the undesirable oscillatory behaviour of the axial intensity of the truncated beams [11–13]. There exist several experiments for generating the nearly non-diffracting fields. The simplest solution to the scalar wave equation, the zero-order Bessel beam, can be obtained by applying an annular ring mask [14], holographic optical element [15, 16], reflective, refractive or diffractive axicons [17–20], aberrated lens [18] or two-element aspherical system [21, 22]. Bessel–Gauss beams can also be generated directly from the laser resonator [23, 24].

Recently, attention has also been concentrated on the vector non-diffractive solutions to the free-space wave equation. Romea and Kimura [25] developed the wave equation on the assumption that the longitudinal electric field resembles the zero-order Bessel function of the first kind. The model resulted in a radially polarized diffraction-free beam whose radial electric field corresponds to the first-order Bessel function of the first kind. The radially polarized non-diffracting beam was also realized experimentally by Tidwel *et al.* [26] and it was suggested that it could be used for accelerating relativistic particles. The azimuthally polarized

† Also with Joint Laboratory of Optics of Palacký University and Physical Institute of Academy of Sciences of the Czech Republic.

non-diffracting beams were examined by Jordan and Hall [27]. Such beams were obtained as solutions to the paraxial wave equation for the azimuthal beam component.

This paper describes the non-diffractive beams as solutions to the vector Helmholtz wave equation. The possibility of realizing such a beam experimentally is explored by applying the vector angular spectrum composed of polarized plane waves. We examine the general solutions providing radially, azimuthally, circularly and linearly polarized Bessel beams of an arbitrary order. The radially and azimuthally polarized beams discussed in [25, 26] can be obtained from our results as special solutions. We also derive expressions for the transverse and longitudinal electric fields of the linearly polarized Bessel beams. The electric field components are circularly non-symmetric and in general cannot be approximated by the scalar solution to the wave equation, as stated [27]. Radial and azimuthal polarizations are examined for the non-diffracting beams produced by axicon polarizers by using initially unpolarized and linearly polarized light, respectively.

2. Concept of the vector non-diffractive beams

2.1. *Solutions to the vector wave equation*

Consider a monochromatic electromagnetic field with the electric vector $\mathbf{E}$ given in the form

$$\mathbf{E}(\mathbf{r}, t) = \tfrac{1}{2}[\mathbf{U}(\mathbf{r}) \exp(-i\omega t) + \mathbf{U}^*(\mathbf{r}) \exp(i\omega t], \tag{1}$$

where $\mathbf{U}$ is a complex vector amplitude. On the assumption that the electromagnetic field propagates through a source-free homogeneous isotropic medium, its vector potential $\mathbf{A}$ can be written as a superposition of the solutions to the vector Helmholtz wave equation [3]

$$\mathbf{A}(\mathbf{r}) = \frac{1}{i\omega} \sum_n (\alpha_n \mathbf{M}_n + \beta_n \mathbf{N}_n + \gamma_n \mathbf{L}_n), \tag{2}$$

where

$$\mathbf{L} = \nabla \Psi,$$

$$\mathbf{M} = \nabla \times \mathbf{u}\Psi,$$

$$\mathbf{N} = \frac{1}{k} \nabla \times \mathbf{M}.$$

The used symbols $\mathbf{u}$ and Ψ denote an arbitrary constant vector and a solution to the scalar Helmholtz wave equation, respectively, and k is the wave number. As the vector potential is assumed to be solenoidal, the vector complex amplitude can be written in the form

$$\mathbf{U}(\mathbf{r}) = \sum_n \alpha_n \mathbf{M}_n + \beta_n \mathbf{N}_n. \tag{3}$$

On the assumption that $\mathbf{u} \equiv \mathbf{u}_z$ is the unit vector in the z-direction and the solutions of the scalar wave equation Ψ_n represent non-diffractive beams,

$$\Psi_n(\mathbf{r}) = J_n(aR) \exp(ibz) \exp(in\varphi), \tag{4}$$

where $a^2 + b^2 = k^2$, and R, φ and z are cylindrical coordinates, the vector wave functions, $\mathbf{M}_n$ and $\mathbf{N}_n$, can be written in the form

$$\mathbf{M}_n = \frac{a}{2}\,[\mathbf{u}_R \mathrm{i}(J_{n-1}(aR) + J_{n+1}(aR)) - \mathbf{u}_\varphi(J_{n-1}(aR) - J_{n+1}(aR))]\exp(\mathrm{i}bz)\exp(\mathrm{i}n\varphi), \tag{5}$$

$$\mathbf{N}_n = \frac{a}{2k}\,[\mathbf{u}_R \mathrm{i}b(J_{n-1} - J_{n+1}) - \mathbf{u}_\varphi b(J_{n-1} + J_{n+1}) + \mathbf{u}_z 2aJ_n]\exp(\mathrm{i}bz)\exp(\mathrm{i}n\varphi), \tag{6}$$

where $\mathbf{u}_R$ and $\mathbf{u}_\varphi$ are unit vectors in the radial and azimuthal directions, respectively, and $J_j \equiv J_j(aR)$ denote jth-order Bessel functions of the first kind. The radial, azimuthal and longitudinal components of the vector amplitude can now be written as

$$U_R = -\mathrm{i}\frac{a}{2}\exp(\mathrm{i}bz)\sum_n \exp(\mathrm{i}n\varphi)(\exp(\mathrm{i}\varphi)\alpha^{(+)}_{n+1} + \exp(-\mathrm{i}\varphi)\alpha^{(-)}_{n-1})J_n(aR), \tag{7}$$

$$U_\varphi = \frac{a}{2}\exp(\mathrm{i}bz)\sum_n \exp(\mathrm{i}n\varphi)(\exp(\mathrm{i}\varphi)\alpha^{(+)}_{n+1} - \exp(-\mathrm{i}\varphi)\alpha^{(-)}_{n-1})J_n(aR), \tag{8}$$

$$U_z = -\frac{a^2}{2b}\exp(\mathrm{i}bz)\sum_n \exp(\mathrm{i}n\varphi)(\alpha^{(+)}_n - \alpha^{(-)}_n)J_n(aR), \tag{9}$$

where

$$\alpha^{(+)}_n = \alpha_n + \frac{b}{k}\beta_n,$$

$$\alpha^{(-)}_n = \alpha_n - \frac{b}{k}\beta_n.$$

Let us now concentrate our attention on the simplest vector non-diffractive beams which can be obtained from the general solution (7–9).

Example 1: Azimuthally polarized beam

$$\alpha^{(+)}_n = \alpha^{(-)}_n = \alpha\delta_{0n}, \left(\mathbf{A} = \frac{\alpha}{\mathrm{i}\omega}\mathbf{M}_0\right):$$

$$U_R = 0,$$

$$U_\varphi = -a\alpha J_1(aR)\exp(\mathrm{i}bz),$$

$$U_z = 0.$$

Example 2: Radially polarized beam

$$\alpha^{(+)}_n = -\alpha^{(-)}_n = \alpha\delta_{0n}, \left(\mathbf{A} = \frac{\alpha k}{\mathrm{i}b\omega}\mathbf{N}_0\right):$$

$$U_R = \mathrm{i}2a\alpha J_1(aR)\exp(\mathrm{i}bz),$$

$$U_\varphi = 0,$$

$$U_z = -\frac{a^2}{b}\alpha J_0(aR)\exp(\mathrm{i}bz).$$

Example 3: Circularly polarized beam

$$\alpha^{(-)}_n = 0, \alpha^{(+)}_n = \alpha\delta_{2n}, \left(\mathbf{A} = \frac{\alpha k}{\mathrm{i}\omega(b+k)}\left(\frac{b}{k}\mathbf{M}_2 + \mathbf{N}_2\right)\right):$$

$$U_x = -\mathrm{i}\frac{\alpha a}{2} J_1(aR)\exp(\mathrm{i}\varphi)\exp(\mathrm{i}bz),$$

$$U_y = \frac{\alpha a}{2} J_1(aR)\exp(\mathrm{i}\varphi)\exp(\mathrm{i}bz),$$

$$U_z = -\frac{\alpha a^2}{2b} J_2(aR)\exp(\mathrm{i}2\varphi)\exp(\mathrm{i}bz).$$

Example 4: Linearly polarized beam

$$\alpha_n^{(+)} = \alpha\delta_{1n}, \alpha_n^{(-)} = \alpha\delta_{-1n}, \left(\mathbf{A} = \frac{\alpha}{\mathrm{i}2\omega}\left(\mathbf{M}_1 + \mathbf{M}_{-1} + \frac{b}{k}(\mathbf{N}_1 - \mathbf{N}_{-1})\right)\right):$$

$$U_x = -\mathrm{i}\alpha a J_0(aR)\exp(\mathrm{i}bz),$$

$$U_y = 0,$$

$$U_z = -\alpha\frac{a^2}{b} J_1(aR)\exp(\mathrm{i}bz)\cos\varphi.$$

2.2. *Decomposition to the vector angular spectrum*

The vector complex amplitude of the electromagnetic field propagating into $z > 0$ can be decomposed to the angular spectrum of the form

$$\mathbf{U}(\mathbf{r}) = \exp(\mathrm{i}kz)\iint_{-\infty}^{\infty} \mathbf{a}(p,q)\exp[-\tfrac{1}{2}\mathrm{i}k(p^2+q^2)z]\exp[\mathrm{i}k(px+qy)]\,\mathrm{d}p\,\mathrm{d}q, \quad (10)$$

where p and q are the direction cosines of the angular components with respect to the x and y axes, respectively, and the direction cosine m, related to the z axis, was approximated by $m \approx 1-(p^2+q^2)/2$. Applying the azimuthal angles Θ and ϕ, the angular spectrum can be rewritten to the form [4]

$$\mathbf{U}(\mathbf{r}) = \frac{\mathrm{i}k}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}\phi\int_0^{\pi/2}\mathrm{d}\Theta\sin\Theta\,\mathbf{F}(\Theta,\phi)\exp(\mathrm{i}k\mathbf{s}\cdot\mathbf{r}), \quad (11)$$

where $\mathbf{s} \equiv \mathbf{s}(p,q,m)$ is the unit direction vector. To obtain the non-diffractive field, the angular spectrum must be related to the Dirac delta-function

$$\mathbf{F}(\Theta,\phi) = \mathbf{V}(\phi)\frac{\delta(\Theta-\Theta_0)}{|\sin\Theta_0|}, \quad (12)$$

where $\Theta_0 < \pi/2$ and $\mathbf{V}(\phi)$ is an arbitrary vector complex function. By using the polar coordinates R and φ, the vector complex amplitude of the non-diffractive field can be written in the form

$$\mathbf{U}(\mathbf{r}) = \frac{\mathrm{i}k}{2\pi}\exp(\mathrm{i}bz)\int_{-\pi}^{\pi}\mathbf{V}(\phi)\exp[\mathrm{i}aR\cos(\phi-\varphi)]\,\mathrm{d}\phi, \quad (13)$$

where

$$a = k\sin\Theta,$$

$$b = k\cos\Theta,$$

$$\Theta \equiv \Theta_0.$$

On the assumption that the field amplitude is the scalar function of the form

$$V = \frac{\mathrm{i}^{-(n+1)}}{k}\exp(\mathrm{i}n\phi), \quad (14)$$

we obtain the known diffraction-free beam transverse intensity distribution which is given by the Bessel functions J_n.

Let us now concentrate our attention on the vector non-diffractive beams which are assumed to be composed of the angular components given by the Dirac delta-function (12). Such angular components are plane waves with their wave vectors $\mathbf{k}_\phi$ on the conical surface. The vector amplitudes of the plane waves $\mathbf{V}'(\phi) \equiv (V'_x, V'_y, V'_z)$ are defined in the Cartesian coordinate system (x', y', z') whose z'-axis coincides with the wave vectors $\mathbf{k}_\phi$. The corresponding vector amplitudes $\mathbf{V}(\phi) \equiv (V_x, V_y, V_z)$ used in equation (13) can be obtained by the matrix transformation

$$\mathbf{V}(\phi) = \mathbf{V}'(\phi)\mathbf{T}, \tag{15}$$

where

$$\mathbf{T} = \begin{pmatrix} -\sin\phi & \cos\phi & 0 \\ -\cos\Theta\cos\phi & -\cos\Theta\sin\phi & \sin\Theta \\ -\sin\Theta\cos\phi & -\sin\Theta\sin\phi & \cos\Theta \end{pmatrix}. \tag{16}$$

If we now define the vector amplitudes $\mathbf{V}'$, the various polarized non-diffracting beams can be simply examined.

3. Radially and azimuthally polarized non-diffractive beams

The simplest solution providing the radially polarized non-diffracting beams can be obtained on the assumption that its angular spectrum is composed of plane waves with constant amplitudes and phases at the given plane $= z = z_0$. In that case, the amplitude components can be written as

$$V'_y = V_0 = \text{const}, \quad V'_x = V'_z = 0. \tag{17}$$

The proposed model is related to the experiment in which unpolarized light is transmitted through axicon polarizers. As stated in [28] this technique can provide both of the nearly pure radial and azimuthal polarizations, respectively, with transmission of the wanted component of more than 96%. On introducing the amplitude transformation (15) into (13), the beam amplitude components can be obtained analytically as

$$U_x(R, \varphi) = -V_0 b \cos\varphi J_1(aR) \exp(ibz), \tag{18}$$

$$U_y(R, \varphi) = -V_0 b \sin\varphi J_1(aR) \exp(ibz), \tag{19}$$

$$U_z(R, \varphi) = iV_0 a J_0(aR) \exp(ibz), \tag{20}$$

where J_0 and J_1 are the zero and first-order Bessel functions of the first kind, respectively. The beam is radially polarized with the radial and longitudinal intensity components given by

$$I_T(R) = \tfrac{1}{2} V_0^2 b^2 J_1^2(aR), \tag{21}$$

$$I_L(R) = \tfrac{1}{2} V_0^2 a^2 J_0^2(aR). \tag{22}$$

The transverse and longitudinal intensity components (see figure 1) depend upon $b = k\cos\Theta$ and $a = k\sin\Theta$, respectively, so that the longitudinal component of the field is appreciable only for beams with a small spot size. Note that the

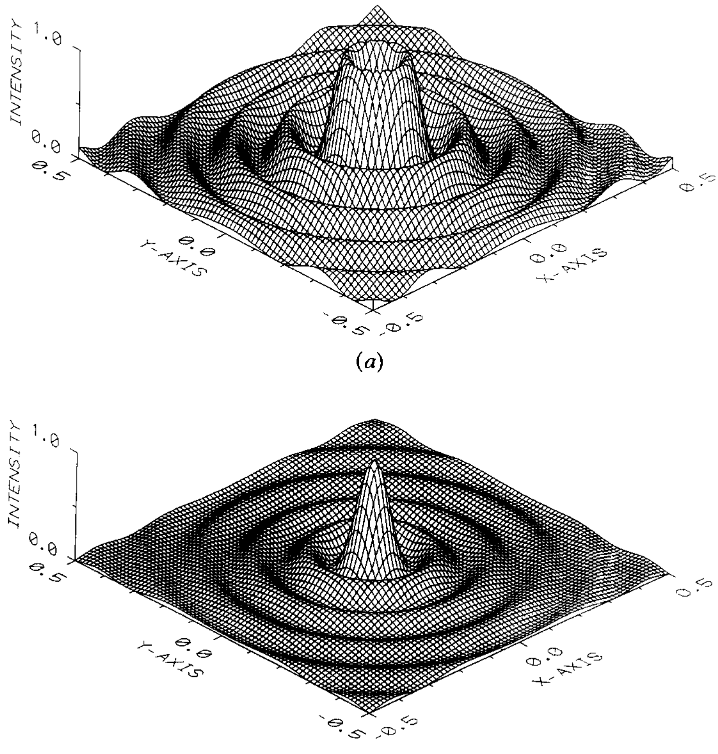


Figure 1. Intensity of the radially polarized non-diffractive beam generated by an axicon polarizer ($n=0$, unpolarized input wave): (*a*) normalized transverse intensity component, (*b*) normalized longitudinal intensity component.

above-examined angular spectrum (12) and (14) with $n=0$ provides the non-diffractive zero-order Bessel beam in the scalar approximation.

Consider now the more general vector beam whose angular spectrum consists of components with phases varying in dependence on the azimuthal angle ϕ. Such a situation is important as the phase variations of the angular components result in high-order Bessel beams in the scalar approximation. The radially polarized beam can now be defined by the amplitudes of the angular components given by

$$V'_y = V_0 \exp(\mathrm{i}n\phi), \quad V'_x = V'_z = 0. \tag{23}$$

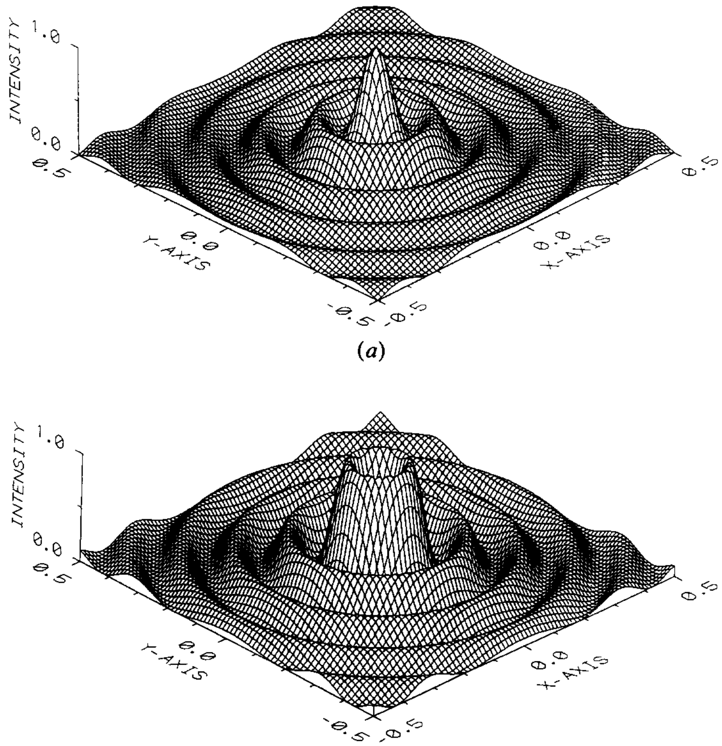
Applying the transformation matrix (16), we obtain the following amplitude components of the formed non-diffractive beam

$$U_x(R,\varphi) = \mathrm{i}^{(n+2)}\tfrac{1}{2}V_0 b \exp(\mathrm{i}n\varphi)[\exp(\mathrm{i}\varphi)J_{n+1}(aR) - \exp(-\mathrm{i}\varphi)J_{n-1}(aR)]\exp(\mathrm{i}bz), \tag{24}$$

$$U_y(R,\varphi) = \mathrm{i}^{(n+1)}\tfrac{1}{2}V_0 b \exp(\mathrm{i}n\varphi)[\exp(\mathrm{i}\varphi)J_{n+1}(aR) + \exp(-\mathrm{i}\varphi)J_{n-1}(aR)]\exp(\mathrm{i}bz), \tag{25}$$

$$U_z(R,\varphi) = \mathrm{i}^{(n+1)}V_0 a \exp(\mathrm{i}n\varphi)J_n(aR)\exp(\mathrm{i}bz). \tag{26}$$

The transverse and longitudinal intensity components can be expressed in the form



(a)

(b)

Figure 2. As in figure 1 ($n = 1$, unpolarized input wave).

$$I_\mathrm{T}(R) = \tfrac{1}{4} V_0^2 b^2 (J_{n-1}^2(aR) + J_{n+1}^2(aR)), \tag{27}$$

$$I_\mathrm{L}(R) = \tfrac{1}{2} V_0^2 a^2 J_n^2(aR). \tag{28}$$

Results obtained numerically for $n = 1$ and $n = 2$, respectively, are shown in figures 2 and 3. Note that the transverse intensity component is non-zero at the axial points of the beam only if $n = \pm 1$. As is obvious, the above-examined non-diffractive beam can be formed from initially unpolarized light. If the examined axicon polarizer providing radial polarization is used with the initially linearly polarized light [28], the amplitudes of the angular components become

$$V_y' = V_0 \cos\phi \exp(\mathrm{i}n\phi), \quad V_x' = V_z' = 0, \tag{29}$$

and the amplitude components of the generated beam are obtained in the forms

$$U_x(R,\varphi) = \mathrm{i}^{(n+1)} \tfrac{1}{2} V_0 b \exp(\mathrm{i}n\varphi)[J_n(aR) - \tfrac{1}{2}(\exp(\mathrm{i}2\varphi) J_{n+2}(aR) + \exp(-\mathrm{i}2\varphi) J_{n-2}(aR))] \exp(\mathrm{i}bz), \tag{30}$$

$$U_y(R,\varphi) = \mathrm{i}^{(n+2)} \tfrac{1}{4} V_0 b \exp(\mathrm{i}n\varphi)[\exp(\mathrm{i}2\varphi) J_{n+2}(aR) - \exp(-\mathrm{i}2\varphi) J_{n-2}(aR)] \exp(\mathrm{i}bz), \tag{31}$$

$$U_z(R,\varphi) = \mathrm{i}^{(n+2)} \tfrac{1}{2} V_0 a \exp(\mathrm{i}n\varphi)[\exp(\mathrm{i}\varphi) J_{n+1}(aR) - \exp(-\mathrm{i}\varphi) J_{n-1}(aR)] \exp(\mathrm{i}bz). \tag{32}$$

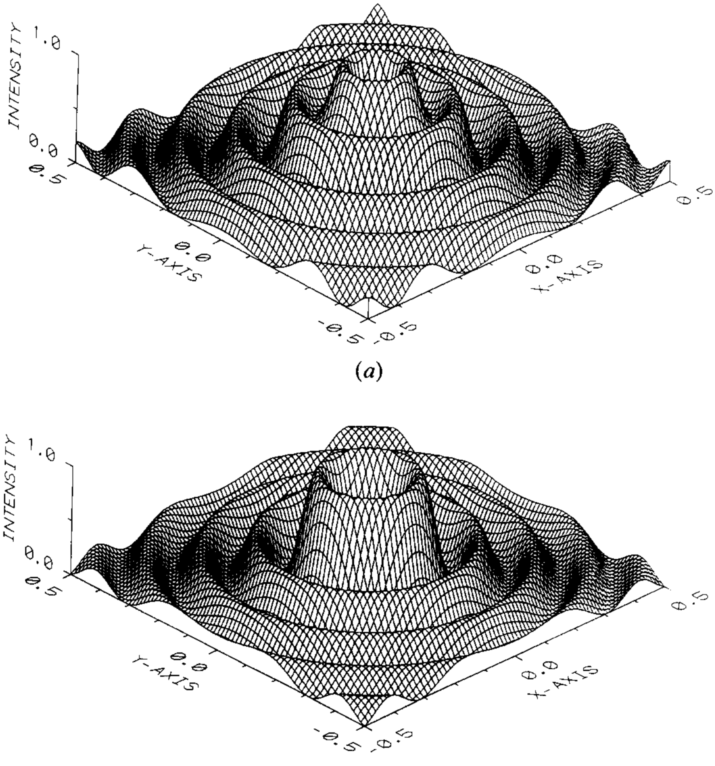


Figure 3. As in figure 1 ($n = 2$, unpolarized input wave).

The circularly non-symmetrical intensity distributions can now be obtained as

$$I_T(R, \varphi) = \tfrac{1}{8} V_0^2 b^2 [J_n^2 + \tfrac{1}{2}(J_{n+2}^2 + J_{n-2}^2) - (J_{n+2} + J_{n-2}) J_n \cos 2\varphi], \quad (33)$$

$$I_L(R, \varphi) = \tfrac{1}{8} V_0^2 a^2 (J_{n+1}^2 + J_{n-1}^2 - 2J_{n+1} J_{n-1} \cos 2\varphi), \quad (34)$$

where $J_j \equiv J_j(aR)$. As is obvious from figures 4 and 5, the intensity distributions are circularly non-symmetrical due to azimuthal amplitude dependence of the angular components. On the assumption that the linearly polarized input wave is transformed by the radially polarizing axicon without azimuthal phase variations, i.e. $n = 0$, the amplitude components of the generated diffraction-free beam can be simplified to the forms

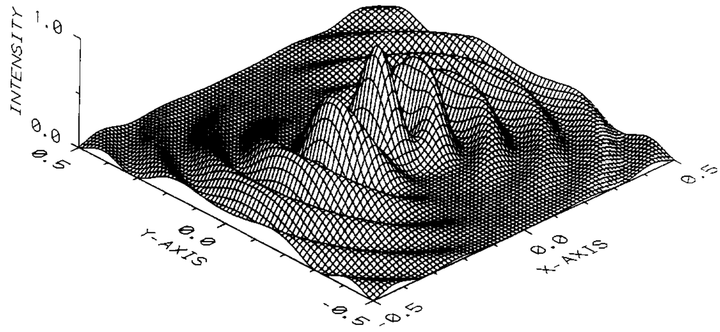
$$U_x(R, \varphi) = \mathrm{i}\tfrac{1}{2} V_0 b [J_0(aR) - \cos 2\varphi J_2(aR)] \exp(ibz), \quad (35)$$

$$U_y(R, \varphi) = -\mathrm{i}\tfrac{1}{2} V_0 b \sin 2\varphi J_2(aR) \exp(\mathrm{i}bz), \quad (36)$$

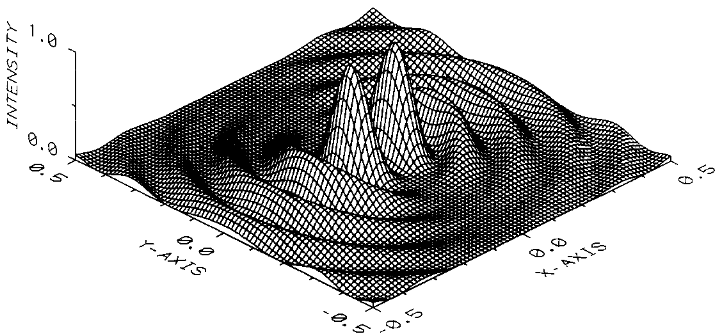
$$U_z(R, \varphi) = -V_0 a \cos \varphi J_1(aR) \exp(\mathrm{i}bz). \quad (37)$$

We now consider the azimuthally polarizing axicon used with the unpolarized and linearly polarized input light, respectively, so that the corresponding amplitudes of the angular components can be written as

$$V'_x = V_0 \exp(\mathrm{i}n\phi), \qquad V'_y = V'_z = 0, \quad (38)$$



(*a*)



(*b*)

Figure 4. As in figure 1 ($n = 0$, linearly polarized input wave).

$$V'_x = V_0 \cos\phi \exp(in\phi), \quad V'_y = V'_z = 0. \tag{39}$$

Applying the transformation matrix (16) we can verify that the amplitudes of the azimuthally and radially polarized beams are simply related as

$$U_x^{(az)}(\mathbf{r}) = -\frac{1}{b} U_y^{(\mathrm{rad})}(\mathbf{r}), \tag{40}$$

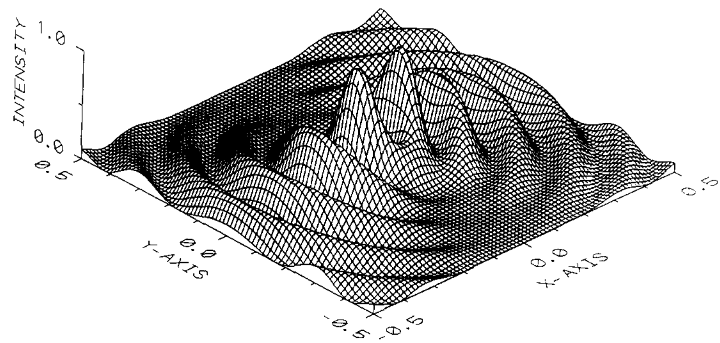
$$U_y^{(az)}(\mathbf{r}) = \frac{1}{b} U_x^{(\mathrm{rad})}(\mathbf{r}), \tag{41}$$

$$U_z^{(az)}(\mathbf{r}) = 0. \tag{42}$$

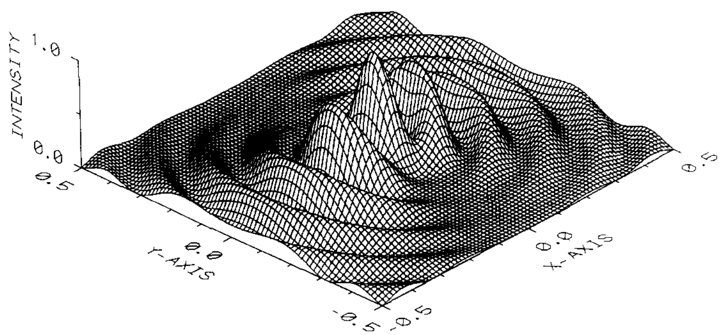
4. Linearly polarized nondiffractive beams

The simplest form of the linearly polarized non-diffracting beam was examined in example 4 as a solution to the vector Helmholtz wave equation. Let us now discuss the possibility of generating such a beam from the linearly polarized wave applying the linear axicon. The examined experimental situation is related to the field amplitudes by

$$V'_x = V_0 \frac{\cos\Theta \cos\phi}{(\cos^2\Theta + \sin^2\Theta \sin^2\phi)^{1/2}} \exp(in\phi), \tag{43}$$



(*a*)



(*b*)

Figure 5. As in figure 1 ($n = 1$, linearly polarized input wave).

$$V'_y = V_0 \frac{\sin\phi}{(\cos^2\Theta + \sin^2\Theta\sin^2\phi)^{1/2}} \exp(\mathrm{i}n\phi), \tag{44}$$

$$V'_z = 0. \tag{45}$$

Applying the transformation matrix (16), the amplitude components of the formed linearly polarized non-diffractive beam can be obtained in the form

$$U_x = 0, \tag{46}$$

$$U_y = \mathrm{i}^{1-n} k V_0 \exp(\mathrm{i}n\varphi)\exp(\mathrm{i}bz) \sum_{q=0}^{\infty} v_q(\exp(-\mathrm{i}2q\varphi) J_{-(2q+n)} + \exp(\mathrm{i}2q\varphi) J_{(2q-n)}), \tag{47}$$

$$U_z = \mathrm{i}^{1-n} \tfrac{1}{2} k V_0 \exp(\mathrm{i}n\varphi)\exp(\mathrm{i}bz)\tan\Theta \sum_{q=0}^{\infty} w_q(\exp[-\mathrm{i}(2q+1)\varphi] J_{(2q-n+1)} + \exp[\mathrm{i}(2q+1)\varphi) J_{-(2q+n+1)}), \tag{48}$$

where

$$v_q = \left(1 - \frac{\delta_{q0}}{1 + (-1)^n}\right) \sum_{m=q}^{\infty} (-1)^m \binom{2m}{m-q} \frac{(2m-1)!!}{m!8^m} \tan^{2m}\Theta,$$

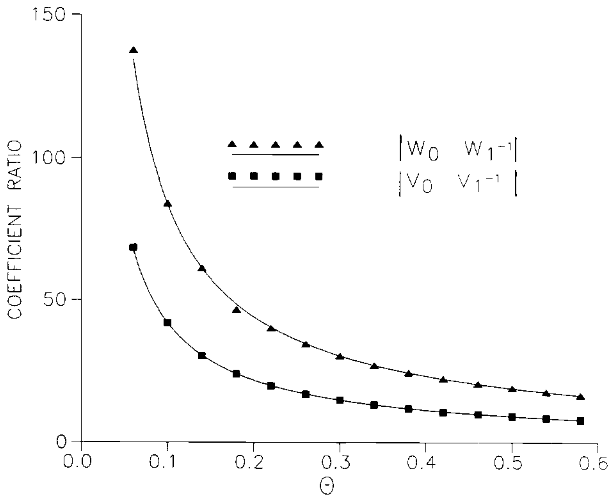


Figure 6. Ratio of the weighting coefficients for the linearly polarized non-diffractive beam.

$$w_q = \sum_{m=q}^{\infty} (-1)^m \binom{2m+1}{m-q} \frac{(2m-1)!!}{m!8^m} \tan^{2m}\Theta,$$

$$(2m-1)!! \equiv 1 \qquad \text{for } m = 0,$$

$$(2m-1)!! \equiv 1\cdot 3\cdot 5 \ldots (2m-1) \qquad \text{for } m \neq 0.$$

On the assumption that the angular plane wave components are superposed without phase changes, i.e. $n = 0$, the simplest form of the realizable linearly polarized beam can be expressed as

$$U_x = 0, \tag{49}$$

$$U_y = \mathrm{i}2kV_0 \exp(\mathrm{i}bz) \sum_{q=0}^{\infty} v_q J_{2q}(aR) \cos 2q\varphi, \tag{50}$$

$$U_z = kV_0 \exp(\mathrm{i}bz) \tan\Theta \sum_{q=0}^{\infty} w_q J_{2q+1}(aR) \sin 2q\varphi. \tag{51}$$

The used coefficients v_q and w_q strongly decrease with increasing q (see figure 6) so that the simplest form of the linearly polarized beam discussed in example 4 represents a good approximation to the one obtained in the experiment only if the beam central spot is much larger in comparison with the wavelength.

5. Conclusions

The amplitude components of the vector non-diffractive beams were obtained as general solutions to the Helmholtz wave equation. The simplest beam forms were related to the radially, azimuthally, circularly and linearly polarized beams. The possibility of realizing such beams was discussed, applying the vector angular spectrum for the experiments based on the axicon polarizers used with both the initially unpolarized and linearly polarized light.

Acknowledgment

This work was partially supported by the Grant No. 202/94/0789 of the Grant Agency of Czech Republic.

References

[1] DURNIN, J. E., 1987, *J. opt. Soc. Am.* A, **4,** 651.
[2] DURNIN, J. E., MICELI, J. J., and EBERLY, J. H., 1987, *Phys. Rev. Lett.*, **54,** 1499.
[3] STRATTON, J. A., 1941, *Electromagnetic Theory* (New York: McGraw-Hill).
[4] M. NIETO-VESPERINAS, M., 1991, *Scattering and Diffraction in Physical Optics* (New York: Wiley).
[5] GORI, F., GUATTARI, G., and PADOVANI, C., 1987, *Optics Commun.*, **64,** 491.
[6] ZAHID, M., and ZUBAIRY, M. S., 1989, *Optics Commun.*,. **70,** 361.
[7] MARON, E., KONFORTI, N., MENDLOVIC, D., and KATZ, J., 1993, in *SPIE Proc. Conf., Optics as a Key to High Technology*, Budapest, Hungary, p. 526.
[8] VICARI, L., 1989, *Optics Commun.*, **70,** 263.
[9] OVERFELT, P. L., and KENNEY, C. S., 1991, *J. opt. Soc. Am.* A, **8,** 732.
[10] BOUCHAL, Z., 1993, *J. mod. Optics*, **40,** 1325.
[11] COX, A. J., and DIBBLE, D C., 1992, *J. opt. Soc. Am.* A, **9,** 282.
[12] COX, A. J., and D'ANNA, J., 1992, *Optics Lett.*, **17,** 232.
[13] HERMAN, R. M., and WIGGINS, T. A., 1992, *Appl. Optics.*, **31,** 5913.
[14] LIN, Y., SEKA, W., EBERLY, J., HUANG, H., and BROWN, D. L., 1992, *Appl. Optics*, **31,** 2708.
[15] TURUNEN, J., VASARA, A., and FRIBERG, A. T., 1988, *Appl. Optics*, **27,** 3959.
[16] COX, A. J., and DIBBLE, D. C., 1991, *Appl. Optics*, **30,** 1330.
[17] PÉREZ, M. V., GOMEZ-REINO, C., and CUADRADO, J. M., 1986, *Optica Acta*, **33,** 1161.
[18] HERMAN, R. M., and WIGGINS, T. A., 1991, *J. opt. Soc. Am.* A, **8,** 932.
[19] ARIMOTO, R., SALOMA, C., TANAKA, T., and KAWATA, S., 1992, *Appl. Optics*, **31,** 6653.
[20] SCOTT, G., and MCARDLE, N., 1992, *Opt. Engng*, **31,** 2640.
[21] THEWES, K., KARIM, M. A., and AWWAL, A. A. S., 1991, *Opt. laser Technol.*, **23,** 105.
[22] IFTEKHARUDDIN, K. M., AWWAL, A. A. S., and KARIM, M. A., 1993, *Appl. Optics*, **32,** 2252.
[23] UEHARA, K., and KIKUCHI, H., 1989, *Appl. Phys.* B, **48,** 125.
[24] JABCZYNSKI, J. K., 1990, *Optics Commun.*, **77,** 292.
[25] ROMEA, R. D., and KIMURA, W. D., 1990, *Phys. Rev.* D, **42,** 1807.
[26] TIDWELL, S. C., FORD, D. H., and KIMURA, W. D., 1992, *Opt. Engng.*, **31,** 1257.
[27] JORDAN, R. H., and HALL, D. G., 1994, *Optics Lett.*, **19,** 427.
[28] SCHAFER, F. P., 1986, *Appl. Phys.* B, **39,** 1.