This assignment deals with:
A) implementation of the 1D Maxwell solver on a spatially (and temporally) staggered grid (Yee method in 1D)
B) examination of the numerical dispersion relation

In the exercises described in our main text, we measured the numerical dispersion of the 1D Maxwell solver with "poor man's" methods without utilizing Fourier transforms. In this assignment you are asked to to use FFT to obtain the spectrum of the simulated solution, and deduce the numerical dispersion from it.

It is OK to utilize any codes found in the previous simulation templates. Note that these programs and scripts come with zero guarantee and may contain bugs.

Each of the following tasks is worth 20% of this homework assignment credit.

Task 1:
Write a simple 1D Maxwell solver along the lines of the example(s) used in the notes and in the class. Apply periodic boundary conditions on both edges of the computational box.

Task 2:
Setup an initial condition with a sharp value of the spatial wavenumber, k, such that the simulated solution will propagate to the right. You may achieve this by setting the electric field to $sin(kx)$, and the magnetic field in the same way discussed in the class. Make sure that you obtain a nice, periodic in space solution, by choosing "admissible" values of k (this depends on the domain size chosen). Demonstrate in a short simulation that your initial condition propagates in the given direction.

Task 3:
During a simulation with a fixed value of k, record the values of the electric field at an arbitrary fixed point in the computational box at every integration step. Calculate the Fourier transform of the recorded history. Inspect the spectrum by plotting its absolute value. It should exhibit two peaks, representing one positive and one negative frequency. Determine the angular frequencies corresponding to these peak positions. This gives you two datums: $[k, \omega_+(k)]$ and $[k, \omega_-(k)]$. The angular frequencies $\omega_\pm$ should be equal in absolute value and have opposite signs. This is because we are simulating a real-valued function, so there are always positive and negative frequency components carrying the same "power".

Note that at this point you have to recall the way FFT routines store their "frequencies."

Task 4:
Repeat the above process (with the exception of plotting the history-spectrum) for values of k spanning the interval of $(-\pi/\Delta x, +\pi/\Delta x)$ and plot both branches $\omega_\pm(k)$. You should obtain numerical dispersion shapes familiar from the class.

Task 5:
Evaluate and plot the error of your measurement by subtracting from $\omega_+(k)$ the analytic formula for the dispersion we have derived in the class. Propose a way to improve the accuracy of your measurement.

Deliverables:

a) your program(s)

b) report in pdf