Crank-Nicolson Method in 1D

This homework assignment concerns the implementation and usage of the Crank-Nicolson method. Your task is to implement it with your own tri-diagonal linear solver.

Note: Functions or objects created in this homework will be re-used when we discuss BPM methods in two transverse dimensions. Write your code with this in mind. In particular, one should identify and separate from each other the implementations of Thomas algorithm for the linear system solver, definition (filling in) of the C-N matrix $L^{(-)}$, and the calculation of the right-hand-side of the system solved at each step.

Note: For simplicity, assume that the computational grid has equidistant spacing.

Deliverables:

A) Implement your own tri-diagonal solver for a system of linear algebraic equation. Take advantage of the fact that no run-time checking should be needed for 'typical' input data in the context of BPM.

B) Create and example with suitable chosen data (say, vector of random numbers) and convince yourself that your implementation works.

C) Implement a C-N based BPM solver for 1D domain with PEC boundary conditions.

D) Use a (one-dimensional!) Gaussian beam propagating at an angle to demonstrate that your solver works correctly. Choose simulation parameters as you see fit.

E) Optional: perform a series of simulation as in D) for several beam propagation angles, and identify the range of angles in which the numerical solution approximates the analytic target within an accuracy you deem to be reasonable.

F) Optional: Implement a modification that allows one to simulate a radially symmetric beam solutions.