Discrete Hankel Transform and its application in BPM

This homework assignment concerns the implementation and usage of the discrete Hankel transform. Your task is to implement a discrete Hankel transform function, test it, and demonstrate its usage in a DHT-BPM method. For this purpose, you are asked to simulate the on-axis intensity of the Poisson‘s bright spot, and compare the outcome to the equivalent simulation utilizing 2D-FFT based algorithm.

Note: It is not necessary to implement calculation of Bessel zeros — one can use the provided tabulation that comes with the exercise-package on Hankel transform.

Deliverables:

A) Create a DHT implementation in a Matlab or any other language program.

B) Test your DHT by demonstrating that the transformation is its own inverse. Optionally, show that Bessel functions with appropriate transverse wavenumbers transform into “delta-functions” in the transverse spatial frequency space.

C) Use DHT-BPM to simulate the bright spot of Arago. Execute this simulation with both, the 2D-FFT code you have developed in the previous homework assignment, and your new DHT-based implementation. Your task is to produce the data for on-axis intensity similar to that discussed in the class. You are free to choose the parameters of this “experiment” as you see fit, but your simulation should capture the behavior in a close vicinity of the screen.

D) Produce a figure, showing the three data sets: A) analytic formula, B) results of your FFT-BPM simulation C) results of your DHT-BPM simulation.

E) Time your simulation runs and present a performance comparison between 2D FFT and DHT based algorithms.

Note: Because DHT-based algorithm takes advantage of the axial symmetry of the problem, it requires only a fraction of grid points, compared to 2D FFT. Thus, you should not need to work with a large DHT matrix...