Purpose: In this assignment, you will build a simple 2D FFT-BPM solver for later use in subsequent exercises.

Task: Reproduce in simulation the profile of an initially collimated Gaussian beam and compare it to the analytic expression (in paraxial approximation).

Unless specified in what follows, you will have to decide what simulation parameters to choose. You should find a regime that will allow to reproduce the expected theoretical curve with a reasonable accuracy.

Deliverables:

1. Implement a function for paraxial Gaussian beam propagation. Assume beam is collimated at $z = 0$, and the function signature is
$$G(x, y, z, w_0, k_0)$$
where x, y are transverse coordinates, z is the propagation distance, $k_0 = 2\pi/\lambda$ is the central-wavelength wavenumber, and w_0 is the beam waist.
Note that you may be able to re-use and extend the 1-dimensional Gaussian beam implementation from a previous exercise.

2. Incorporate the above into your FFT-BPM code to set initial condition
$$A_{in} = G(x, y, z = 0, w_0 = 5\mu\text{m}, k_0) \ ,$$
with lambda = 800 nm.

3. Incorporate the Gaussian formula into your FFT-BPM code to define the analytic solution at the total propagation distance of 2mm:
$$A_{target} = G(x, y, z_{total} = 2\text{mm}) \ .$$

4. Write (optionally starting from the 1D examples discussed in the class) a 2D FFT-BPM solver. With the future application in mind, code for the possibility to propagate in both paraxial and non-paraxial regimes. Run your FFT-BPM to evolve the initial condition into final numerical solution
$$A_{fin}(z_{total}) \ .$$

5. Plot $Re(A_{fin})$ as a 2D map across the transverse dimensions of the computational domain. Sanity check your simulation by showing that the corresponding plot for the target solution is indeed similar. Remember that one has to include or not include the carrier in both the target and simulated solutions for this comparison.

6. Plot $Abs(A_{fin}(x, y = 0))$ (versus x) and that of the target solution in the same plot.

7. Identify a region in which the deviations between the numerical and analytic solution is greatest. EXPLAIN why is the error biggest in that region. Note that the answer to this question may depend on the particular parameter set you will choose for your simulation.

Submit a single tar or zipped folder containing all files necessary for the instructor to re-run and reproduce your simulation. Do not use spaces in file-names.