OPTI-310. Review of useful formulas III.

Huygens-Fresnel principle expressed in terms of an integral over aperture:

$$U(X,Y,Z) \approx \iint_A dydz U_0(0,y,z)\frac{e^{+ikr}}{r} \quad , \quad r = \sqrt{X^2 + (y-Y)^2 + (z-Z)^2}$$

where X stands for distance between the screen and observation plane. y, z are integration variables spanning the aperture, Y, Z are coordinates of the observation point.

Rayleigh-Somerfeld diffraction formula:

$$U_{RS}(X,Y,Z) = -\frac{i}{\lambda}\iint_A dydz U_0(0,y,z)\frac{e^{+ikr}}{r}\cos\alpha$$

Fresnel-Kirchhoff difraction formula:

$$U_{FK}(X,Y,Z) = -\frac{i}{\lambda}\iint_A dydz U_0(0,y,z)\frac{e^{+ikr}}{r}\frac{1}{2}(\cos\beta + \cos\alpha)$$

α and β above are angles between the aperture (surface) normal and line of observation and line of illumination, respectively.
Common approximations - small angles, and "constant" denominator r:

$$R >> \sqrt{y^2+z^2}, \sqrt{Y^2+Z^2} \quad , \quad \frac{1}{r} \approx \frac{1}{R}$$

Approximation (for r in the exponential phase factor):

$$r = \sqrt{X^2 + (y-Y)^2 + (z-Z)^2} \approx R - \frac{yY+zZ}{R} + \frac{y^2+z^2}{2R} + \dots \quad , \quad R = \sqrt{X^2+Y^2+Z^2}$$

Fraunhofer regime:

$$k\frac{y^2+z^2}{2R} = \frac{2\pi}{\lambda}\frac{y^2+z^2}{2R} < \pi$$

Fresnel region:

$$k\frac{y^2+z^2}{2R} = \frac{2\pi}{\lambda}\frac{y^2+z^2}{2R} > \pi$$

Fresnel #:

$$N_F = \frac{y^2+z^2}{\lambda R}$$

Fraunhofer diffraction = Fourier Transform:

$$U_F(X,Y,Z) \approx \iint_A dydz U_0(0,y,z)\exp\left[-ik\frac{yY+zZ}{R}\right] = \iint_A dydz U_0(0,y,z)\exp\left[-i(k_y y + k_z z)\right]$$

$$k_y = kY/R \quad , \quad k_z = kZ/R$$

Single slit diffraction (width b):

$$I(x) = I(0)\frac{\sin^2\beta}{\beta^2} \quad , \quad \beta = \frac{kb}{2}\sin\theta \approx \frac{kb}{2}\theta$$

Double slit diffraction (width b, distance a):

$$E(\theta) = 2E_L\frac{be^{ikR}}{R}\mathrm{sinc}(\beta)\cos(\alpha) \quad , \quad \beta = \frac{kb}{2}\sin\theta \quad , \quad \alpha = \frac{ka}{2}\sin\theta$$

$$I(\theta) = I(\theta = 0)\mathrm{sinc}^2(\beta)\cos^2(\alpha)$$

N-slit diffraction (width b, distance a):

$$I(\theta) = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

Rectangular aperture:

$$I(Y,Z) = I(0)\mathrm{sinc}^2(\alpha')\mathrm{sinc}^2(\beta')$$

$$\alpha' = \frac{kaZ}{2R} \quad , \quad \beta' = \frac{kbY}{2R}$$

Circular aperture:

$$I(q) = I(0)\left[\frac{2J_1(kaq/R)}{kaq/R}\right]^2$$

First (nontrivial) zero of J_1: 3.83

Airy disk radius:

$$q_1 = 1.22\frac{R\lambda}{2a} = 1.22\frac{f\lambda}{D}$$

Fresnel zones:

Outer radius:

$$R_n = \sqrt{\lambda n L}$$

On-axis illumination:

$$|U_p| = |U_1| - |U_2| + |U_3| - |U_4| + |U_5| + \dots$$

$$|U_p| = |U_1|/2 + (|U_1|/2 - |U_2| + |U_3|/2) + (|U_3|/2 - |U_4| + |U_5|/2) + \dots \approx |U_1|/2$$

Fabry-Perot interferometer:

$$I_T = I_0 \frac{T^2}{1 + R^2 - 2R\cos\Delta} = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2(\Delta/2)}$$

$$\Delta = \delta + \delta_r \quad , \quad \delta = 2kd\cos\Theta = \frac{4\pi nd}{\lambda}\cos\Theta$$

Coefficient of finesse:

$$F = \frac{4R}{(1-R)^2}$$

Max and min transmission:

$$A + T + R = 1$$

$$I_{max}/I_0 = \frac{T^2}{(1-R)^2} = \left(\frac{1-A-R}{1-R}\right)^2$$

$$I_{min}/I_0 = \frac{T^2}{(1+R)^2}$$

FWHM of fringes:

$$\delta\Delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}} = \frac{2\pi}{\mathcal{F}}$$

Finesse:

$$\mathcal{F} \equiv \frac{2\pi}{\delta\Delta} = \frac{\pi}{2}\sqrt{F} = \pi\frac{\sqrt{R}}{1-R}$$

Free spectral range = separation between adjacent orders of interference:

$$\nu_F = \frac{c}{2nd}$$

Fabry-Perot cavity modes:

$$\nu_{n+1} - \nu_n = \frac{c}{2nd} = \nu_F$$

Resolving power:

$$RP = \frac{\omega}{\delta\omega} = \frac{\nu}{\delta\nu} = \frac{\lambda}{\delta\lambda}$$

$$RP = N\mathcal{F}$$

Single-layer coating reflectivity (air, n_1, n_T):

$$r = \frac{n_1(1 - n_T)\cos kl - i(n_t - n_1^2)\sin kl}{n_1(1 + n_T)\cos kl - i(n_t + n_1^2)\sin kl}$$

Quater-wave film ($kl = \pi/2$):

$$R = |r|^2 = \frac{(n_T - n_1^2)^2}{(n_t + n_1^2)^2} \quad , \quad \text{zero if} \quad n_1 = \sqrt{n_t}$$

High-reflectivity multi-layer (in air, high low refracive indices n_H, n_L, N high-low layers):

$$R = |r|^2 = \left[\frac{(n_h/n_L)^{2N} - 1}{(n_h/n_L)^{2N} + 1}\right]^2$$