8. Polarization (Hecht 8, Fowles 2.3-2.5)

So far we have considered linear polarization. Goal now is to isolate the polarization state of light, study states of polz'n, and how they are transformed by optical elements.

Consider a harmonic plane-wave propagating along the z-axis

$$\vec{E}(\vec{r},t) = \hat{x}E_x(z,t) + \hat{y}E_y(z,t)$$

$$E_x(z,t) = A_x \cos(kz - \omega t + \phi_x)$$
$$= \frac{1}{2}\left[\mathcal{E}_x e^{i(kz-\omega t)} + c.c\right]$$

$$E_y(z,t) = A_y \cos(kz - \omega t + \phi_y)$$
$$= \frac{1}{2}\left[\mathcal{E}_y e^{i(kz-\omega t)} + c.c\right]$$

with complex amplitudes

$$\mathcal{E}_x = A_x e^{i\phi_x}, \quad \mathcal{E}_y = A_y e^{i\phi_y}$$

Then by rearranging ($\tilde{\tau} = \omega t - kz$)

$$\frac{E_x(\tau)}{A_x} = \cos\tilde{\tau}\cdot\cos\phi_x + \sin\tilde{\tau}\cdot\sin\phi_x$$

$$\frac{E_y(\tilde{\tau})}{A_y} = \cos\tilde{\tau}\cdot\cos\phi_y + \sin\tilde{\tau}\cdot\sin\phi_y$$

Manipulate to get <u>equation of ellipse</u>.

$$\left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2\left(\frac{E_x}{A_x}\right)\left(\frac{E_y}{A_y}\right)\cos\phi_{yx}$$

$$= \sin^2\phi_{yx}$$

$$\phi_{yx} = \phi_y - \phi_x = -\delta$$

Tip of the E-field vector in $(x-y)$ plane must fall on above ellipse parametrically with $\tilde{\tau}$!

Examples are shown in the Figure: Orientation & ellipticity of state of light depend on (A_x/A_y) & ϕ_{yx}.

<u>Examples:</u> (see Applet on webpage)

- $\phi_{yx} = 0$ gives $(E_x/A_x) = (E_y/A_y)$.

or

$$E_y = \left(\frac{A_y}{A_x}\right) E_x \, ,$$

→ straight line of slope (A_y/A_x)

Same for $\phi_{yx} = \pi$ except slope $-(A_y/A_x)$

→ <u>linear</u> polarization

Motion of tip of $\vec{E}(\tau)$ field is a straight line. in (x-y) plane.

- $\phi_{yx} = 0, \pm\pi$, linear polarization
  eg. $\phi_x = \phi_y = 0$

$$\vec{E}(\vec{r},t) = (\hat{x}A_x + \hat{y}A_y)\cos(\tau)$$

A_x & A_y real, this is a linearly polarized state of light.

$A_y = 0$, $\hat{x}$ polarized
$A_x = 0$, $\hat{y}$ polarized
$\Big\}$ $\hat{e}_x = \hat{x}$, $\hat{e}_y = \hat{y}$ Orthonormal polarization states

- $\phi_{yx} \neq 0, \pi$, <u>elliptical polariz'n states</u>

$0 < \phi_{yx} < \pi$ — <u>left-handed ellip.</u>

$\pi < \phi_{yx} < 2\pi$ — <u>right handed.</u>

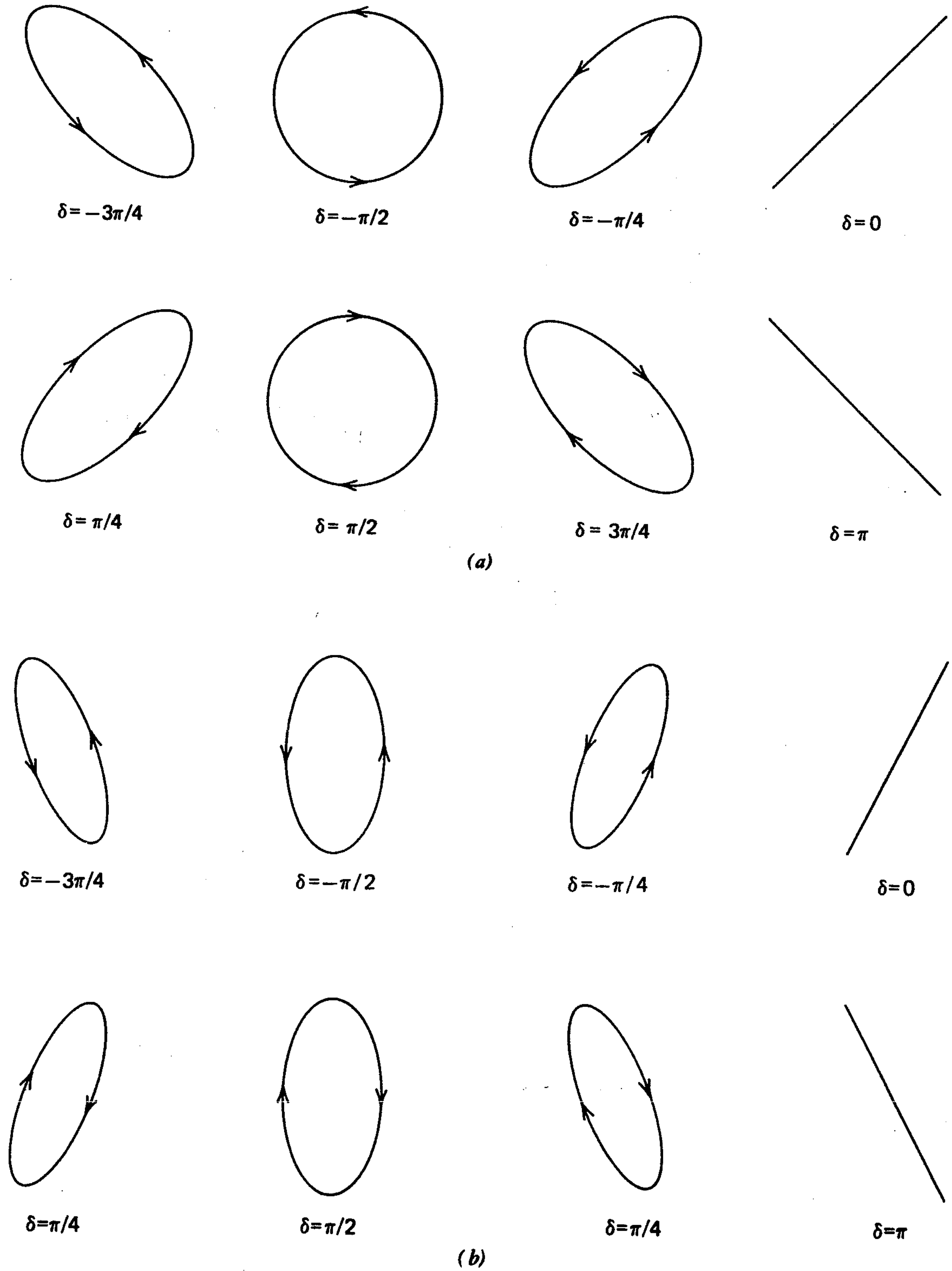


Figure 3.2. Polarization ellipses at various phase angles δ, where: (a) $E_x = \cos(\omega t - kz)$, $E_y = \cos(\omega t - kz + \delta)$; (b) $E_x = \frac{1}{2}\cos(\omega t - kz)$, $E_y = \cos(\omega t - kz + \delta)$.

$$-\delta \equiv \phi_{yx}$$

$$(a)\ \left|\frac{A_y}{A_x}\right| = 1,\quad (b)\ \left|\frac{A_y}{A_x}\right| = 2$$

Table 3.1. Relations between Angular Momentum and Handedness of Circularly Polarized Light

For example, $A_x = A_y = A$, $\phi_x = 0$,
$\boxed{\phi_y = \pi/2}$, $\phi_{yx} = \pi/2$

$$\vec{E}(\vec{r},t) = A\left[\hat{x}\cos(\tau) + \hat{y}\sin(\tau)\right]$$

Tip of $\vec{E}$ field follows a circle!

$\rightarrow$ LHC light $\leftarrow$ counter-clockwise looking along $-z$

$\boxed{\phi_{yx} = 3\pi/2}$ (p. 29-30 Fowles)

$$\vec{E}(\vec{r},t) = A\left[\hat{x}\cos(\tau) - \hat{y}\sin(\tau)\right]$$

$\rightarrow$ RHC light $\leftarrow$ Clockwise looking along $-z$

Also $\phi_{yx} = \pi/2$

$$\vec{E}(\vec{r},t) = \frac{A}{2}\left[\hat{x}\left(e^{i\tau} + e^{-i\tau}\right) - i\hat{y}\left(e^{i\tau} - e^{-i\tau}\right)\right]$$

$$= \frac{A}{\sqrt{2}}\left[\underbrace{\frac{(\hat{x}+i\hat{y})}{\sqrt{2}} e^{i(kz-\omega t)}} + c.c\right]$$

$$= \frac{A}{\sqrt{2}}\left[\hat{e}_- e^{i(kz-\omega t)} + c.c\right]$$

$$\hat{e}_\pm = \frac{1}{\sqrt{2}}(\hat{x} \mp i\hat{y})$$

→ RHC (+) & LHC (−) basis vectors. – here complex!

$$\boxed{\hat{e}_\mu \cdot \hat{e}_\nu^* = \delta_{\mu\nu}}$$

<u>General</u>: two linearly independent polarization states for give direction of propagation (here z).

<u>They form an orthonormal basis</u>

Use linearly polarized basis $\hat{e}_{x,y}$

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left[\hat{x}\,\mathcal{E}_x(z)e^{i(kz-\omega t)} + c.c\right.$$

$$\left. + \hat{y}\,\mathcal{E}_y(z)e^{i(kz-\omega t)} + c.c\right]$$

complex.

$\mathcal{E}_{x,y}(z)$ can account for varying polzn' state, with respect of carrier variation $\exp[i(kz-\omega t)]$

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left[(\hat{x}\,\mathcal{E}_x + \hat{y}\,\mathcal{E}_y)e^{i(kz-\omega t)} + c.c\right]$$

Time averaged Poynting vector is

$$\vec{S} = \frac{1}{2}\epsilon_0 n c\left(|\mathcal{E}_x|^2 + |\mathcal{E}_y|^2\right)\hat{z}.$$

Jones Vectors and matrices

State of polz'n implied in complex amplitudes $\mathcal{E}_x$ and $\mathcal{E}_y$ (positive freqe.)

$$\phi_{yx} = \arg(\mathcal{E}_y/\mathcal{E}_x), \; (A_y/A_x) = \left|\frac{\mathcal{E}_y}{\mathcal{E}_x}\right|.$$

Form Jones vector $\bar{\mathcal{E}}(z)$

$$\bar{\mathcal{E}}(z) = \begin{pmatrix} \mathcal{E}_x(z) \\ \mathcal{E}_y(z) \end{pmatrix} = \mathrm{col}(\mathcal{E}_x(z), \mathcal{E}_y(z))$$

Then Jones vectors for various polz'n states are:

$\bar{\mathcal{E}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ — x polarized

$\bar{\mathcal{E}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ — y polarized

(normally multiplied by $\mathcal{E}_0$)

$\bar{\mathcal{E}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ — RHC

$\bar{\mathcal{E}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}$ — LHC . . .

$$\bar{\mathcal{E}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \rightarrow 45^\circ \text{ linear pol}z\text{n.}$$

$$\bar{\mathcal{E}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \rightarrow -45^\circ \text{ linear } \text{''}.$$

<u>Orthogonal</u> polz'n states have

$$(\bar{\mathcal{E}}_\mu^*)^T \cdot \bar{\mathcal{E}}_\nu = 0, \quad \mu \neq \nu$$

T – <u>transpose</u> $\bar{\mathcal{E}}^T = (\mathcal{E}_x \;\; \mathcal{E}_y)$.

<u>Optical elements</u> (see Applet)

Action of optical elements on the polz'n state via Jones vectors.

- Linear polarizers – eg. removal of x-component of field.

$$M\begin{pmatrix}\mathcal{E}_x\\ \mathcal{E}_y\end{pmatrix} = \begin{pmatrix}0\\ \mathcal{E}_y\end{pmatrix}$$

M - Jones Matrix

(2x2) matrix rep. of M — <u>Jones matrix</u> for element (eg free-space)

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

For transmission axis horizontal (x)

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{(x-polarizer)}$$

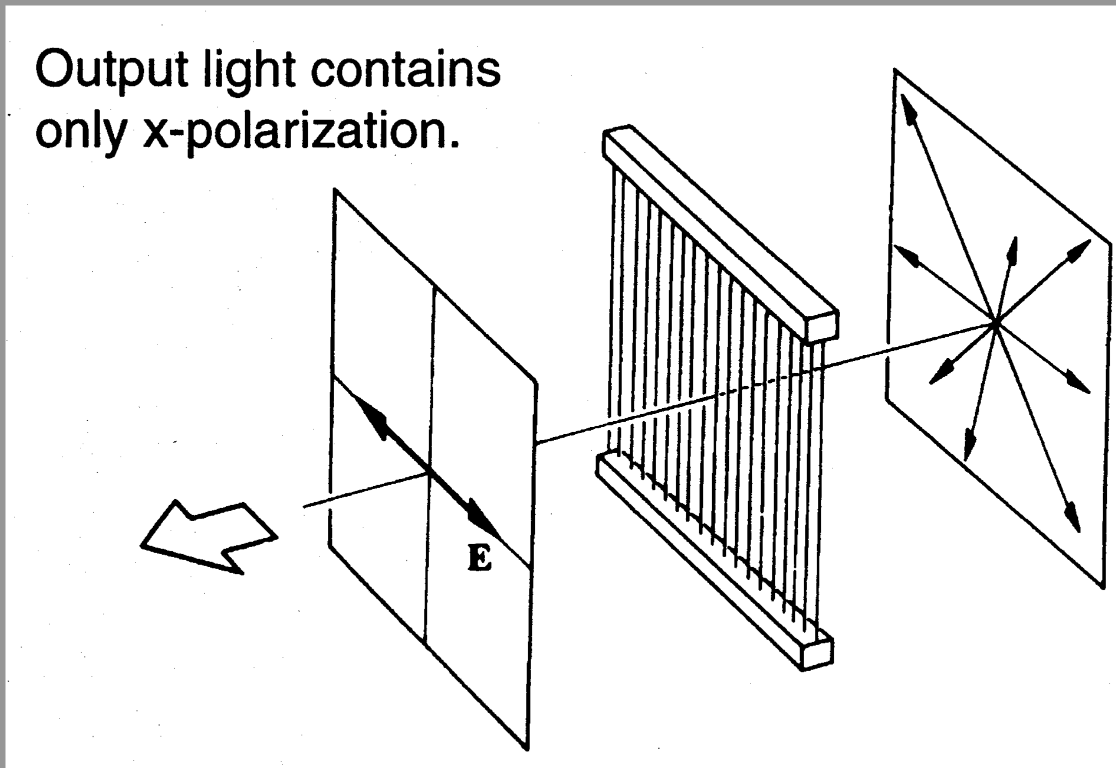
for transmission axis vertical (y)

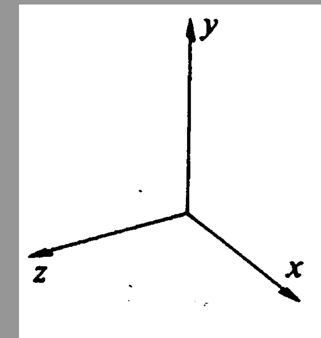
$$M = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad \text{(y-polarizer)}$$

Examples of structures that can act as polarizers: (Hecht 8.2)

- Wire grid polarizer (p. 333 Hecht)
- Polaroid p. 335 (Hecht)
- Anisotropic crystals
- Brewster or polarizing angle (Hecht 348)

Wire Grid Polarizer





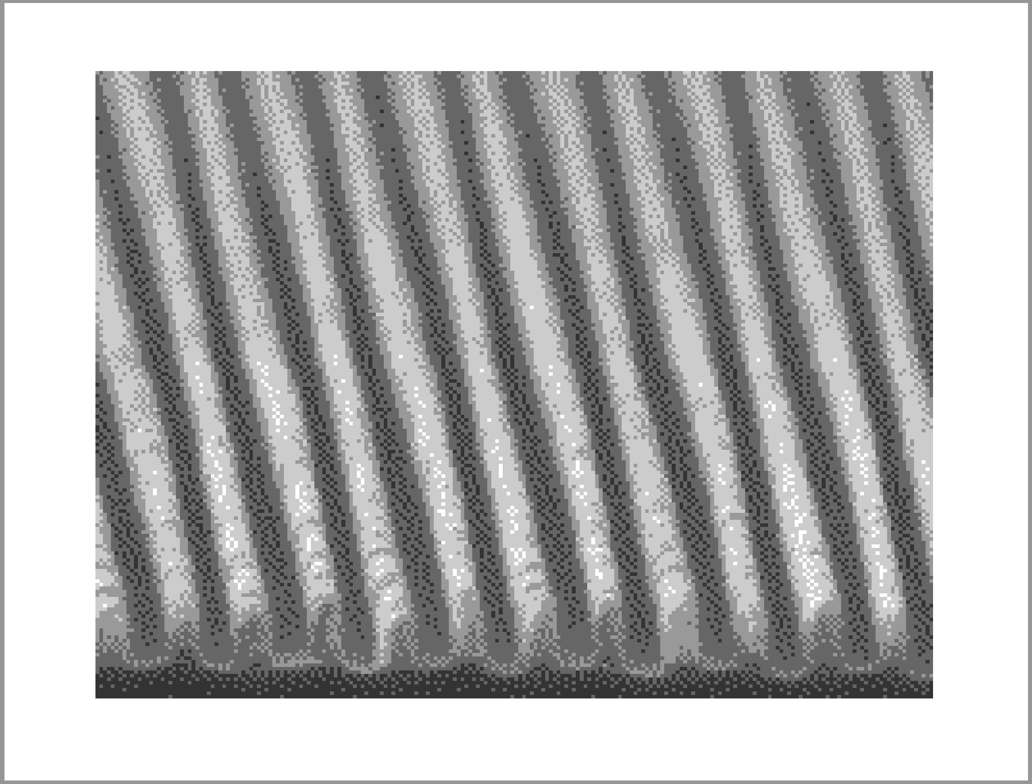
Input light contains both polarizations

The light can excite electrons to move along the wires, which then emit light that cancels the input light. This cannot happen perpendicular to the wires. Such polarizers work best in the IR.

Polaroid sheet polarizers use the same idea, but with long polymers.

Wire grid polarizer in the visible

Using semiconductor fabrication techniques, a wire-grid polarizer was recently developed for the visible.



The spacing is less than 1 micron.

Table 2.1. Jones Matrices for Some Linear Optical Elements

Optical Element		*Jones Matrix*
Linear polarizer	Transmission axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
	Transmission axis vertical	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
	Transmission axis at $\pm 45°$	$\frac{1}{2}\begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$
Quarter-wave plate	Fast axis vertical	$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
	Fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
	Fast axis at $\pm 45°$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \pm i \\ \pm i & 1 \end{bmatrix}$
Half-wave plate	Fast axis either vertical or horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Isotropic phase retarder		$\begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix}$
Relative phase changer		$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$
Circular polarizer	Right	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
	Left	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

Note: Normalization factors are included in the table. These factors are necessary for energy considerations only and can be omitted in calculations concerned primarily with type of polarization. Also, the signs of all matrix elements containing the factor i should be changed if one uses the wave function $\exp i(\omega t - kz)$ rather than $\exp i(kz - \omega t)$.

Rotated optical elements.

Consider a Jones vector $\bar{\mathcal{E}} = \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$ in a rotated basis system $\hat{e}_{x'}, \hat{e}_{y'}$

$$\begin{pmatrix} \mathcal{E}_{x'} \\ \mathcal{E}_{y'} \end{pmatrix} = R(\theta) \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = R^{-1}(-\theta)$$

$R(\theta)$ is orthogonal (transpose = inverse)

Take

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}_L = M \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}_0$$

or in the rotated frame,

$$\begin{pmatrix} \mathcal{E}_{x'} \\ \mathcal{E}_{y'} \end{pmatrix}_L = \underbrace{R(\theta) M R^{-1}(\theta)}_{M'} \overbrace{R(\theta) \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}_0}^{\begin{pmatrix} \mathcal{E}_{x'} \\ \mathcal{E}_{y'} \end{pmatrix}_0}$$

So the Jones matrix in the rotated system is

$$M' = R(\theta) M R^{-1}(\theta)$$

Assume M' is a given element, then back in the lab frame

$$M = R^{-1}(\theta) M' R(\theta)$$
$$= R(-\theta) M' R(\theta).$$

Example: Take M' as a horizontal polarizer $M' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, then

$$M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{pmatrix}$$

- If we set $\theta = \pm 45° = \pm \pi/4$ we get

$$M = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$

see table of Jones matrices on p. 35 of Fowles.

- Consider an incident x-polarized field $\bar{\mathcal{E}}(0) = \mathcal{E}_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, then after the rotated polarizer

$$\bar{\mathcal{E}}(L) = M \bar{\mathcal{E}}(0), \text{ or}$$

$$\begin{pmatrix} \mathcal{E}_x(L) \\ \mathcal{E}_y(L) \end{pmatrix} = \mathcal{E}_0 \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \mathcal{E}_0 \cos^2\theta \\ \mathcal{E}_0 \sin\theta\cos\theta \end{pmatrix}$$

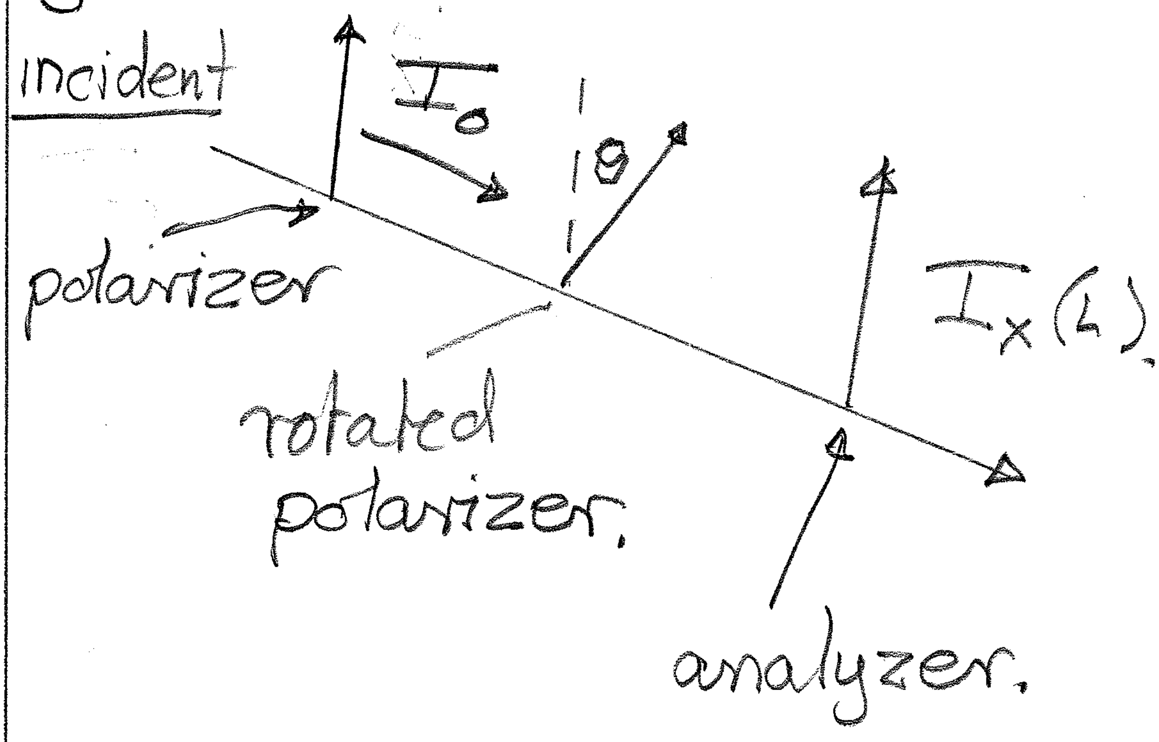
so the field along the x-direction is

$$\mathcal{E}_x(L) = \mathcal{E}_0 \cos^2\theta$$

or in terms of intensity

$$\boxed{I_x(L) = I_0 \cos^4\theta}$$

this can be used to vary the intensity of a laser beam.



We shall look at a variant of this later.

If the initial field was instead polarized along the y-axis we find (try for yourself)

$$I_x(L) = I_0 \cos^2\theta \sin^2\theta \quad - y \text{ pol.}$$
$$I_x(L) = I_0 \cos^4\theta \quad - x \text{ pol.}$$

the average of these is what one gets for initially <u>unpolarized</u> light

$$I_x(L) = \left[I_0 \cos^4\theta + I_0 \cos^2\theta \sin^2\theta\right]/2$$

$$= I_0 \cos^2\theta(\cos^2\theta + \sin^2\theta)/2$$

$$= \frac{I_0}{2}\cos^2\theta$$

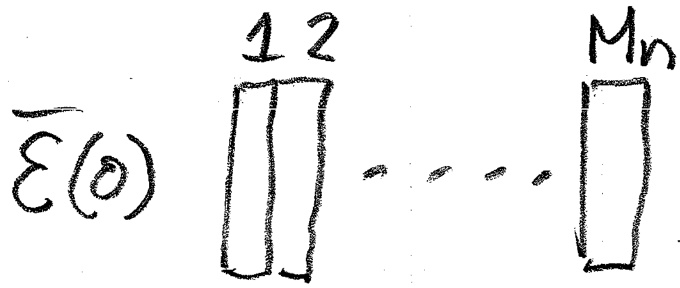
This is called Malus's law. (see Hecht p. 333)

Table 2.1 on p. 35 of Fowles shows a list of common Jones matrices.
We shall explore the so called Jones Calculus

Jones calculus

For lossless elements M is unitary, conserves Poynting vector. Lossy elements, eg polarizers, non-unitary.
Can concatonate Jones matrices for optical systems

$$M = M_n \cdot M_{n-1} \cdots M_1$$



$\bar{\mathcal{E}}(L) = M\,\bar{\mathcal{E}}(0)$

Eg. cross polarizers

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}!$$

So any field is extinguished following a pair of crossed polarizers.

In general once we specify the optical system via $\underline{M}$ and the input polz'n using $\bar{\mathcal{E}}(0)$, we can get $\bar{\mathcal{E}}(L)$ at the output. Thus, we can calculate

$$\phi_{yx}(L) = \arg\left(\frac{\mathcal{E}_y(L)}{\mathcal{E}_x(L)}\right)$$

$$\left(\frac{A_y}{A_x}\right)_L = \left|\frac{\mathcal{E}_y(L)}{\mathcal{E}_x(L)}\right|$$

so we know the nature of the polz'n at the output given $\phi_{yx}(0)$, $(A_y/A_x)_0$ at the input.

This is the idea of the Jones calculus, see Sec. 2.5 of Fowles.

Example: Consider a LHC polarized field incident on a rotated polarizer

$$\begin{pmatrix} \mathcal{E}_x(L) \\ \mathcal{E}_y(L) \end{pmatrix} = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \cos^2\theta + i\sin\theta\cos\theta \\ \sin\theta\cos\theta + i\sin^2\theta \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\theta\, e^{i\theta} \\ \sin\theta\, e^{i\theta} \end{pmatrix} = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}.$$

so

$$\phi_{yx}(L) = 0 \text{ or } \pi.$$

$$\left(\frac{A_y}{A_x}\right)_L = |\tan\theta|$$

$\theta = 0$, LHC $\rightarrow$ linear x-polarized

$\theta = \pi/4$, LHC $\rightarrow$ linear @ 45° etc.

Circular $\longrightarrow$ linear.

Phase retarders & waveplates

Here we shall investigte polarization elements that can produce elliptical from linear polz'n.

Consider the case that x & y polarized fields experience different phase-shifts upon traversing the medium

$$\begin{pmatrix} \mathcal{E}_x(L) \\ \mathcal{E}_y(L) \end{pmatrix} = \begin{pmatrix} \mathcal{E}_x(0) e^{i\phi_x} \\ \mathcal{E}_y(0) e^{i\phi_y} \end{pmatrix} = \bar{\mathcal{E}}(L)$$

$$= M \bar{\mathcal{E}}(0) = \begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix} \begin{pmatrix} \mathcal{E}_x(0) \\ \mathcal{E}_y(0) \end{pmatrix}$$

Hence, if $\phi_x \neq \phi_y$ one field component is retareded in phase w.r.t the other, example of a phase retarder

$$M = e^{i\phi_x} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{pmatrix}$$

$\Delta\phi = \phi_y - \phi_x$. We normally write the Jones matrix for phase retarder as

$$M = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{pmatrix}.$$

This is realized using optical birefringence is anisotropic crystals where n_x & n_y can be different

$$\phi_x = \frac{2\pi}{\lambda} n_x L \text{ — } x\text{-polarized}$$

$$\phi_y = \frac{2\pi}{\lambda} n_y L \text{ — } y\text{-polarized}$$

$$\Delta\phi = \frac{2\pi}{\lambda}(n_y - n_x)L.$$

$\Delta\phi = \pm\pi/2$ – quarter wave plate

$\Delta\phi = \pi$ — half-wave plate.

$\Delta\phi = 2\pi$ — full-wave plate.

- Half-wave plate.

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- Quarter-wave plate

$$M = \begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix}.$$

(+) case means $n_y > n_x$, so fast axis x-axis or horizontal

(-) case, fast axis vertical or y.

> Note: Jones matrices for phase-retarders are defined with respect to crystal axes

- Full-wave plate.

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Transformation of polarization.

We now go through some very important examples of transformation of polarization

- Quarter-wave plates:

Choose linear polarization @ 45° for the incident field

$$\bar{\mathcal{E}}(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

then

$$\bar{\mathcal{E}}(L) = \begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix} \bar{\mathcal{E}}(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}.$$

which is circularly polarized!!

LHC for (+) or fast axis horizontal
RHC " (−) " " " vertical.

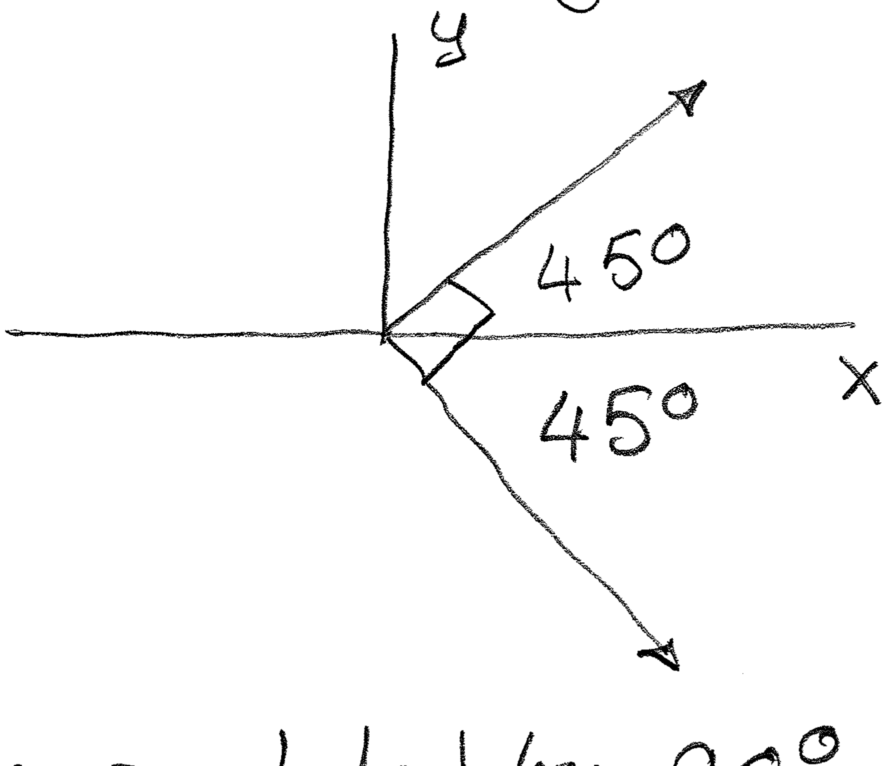
Linear ⟶ circular.

- Half-wave plate.

Again choose input polarization @ 45°, then

$$\bar{\mathcal{E}}(L) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \bar{\mathcal{E}}(0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

which is linear @ -45°, so there is a net rotation of 90°



Linear rotated by 90°

- Phase retarder

Consider LHC polarized light input to a phase retarder

$$\bar{\mathcal{E}}(L) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(\pi/2+\Delta\phi)} \end{pmatrix}$$

Then

$$\phi_{yx}(L) = \frac{\pi}{2} + \Delta\phi$$

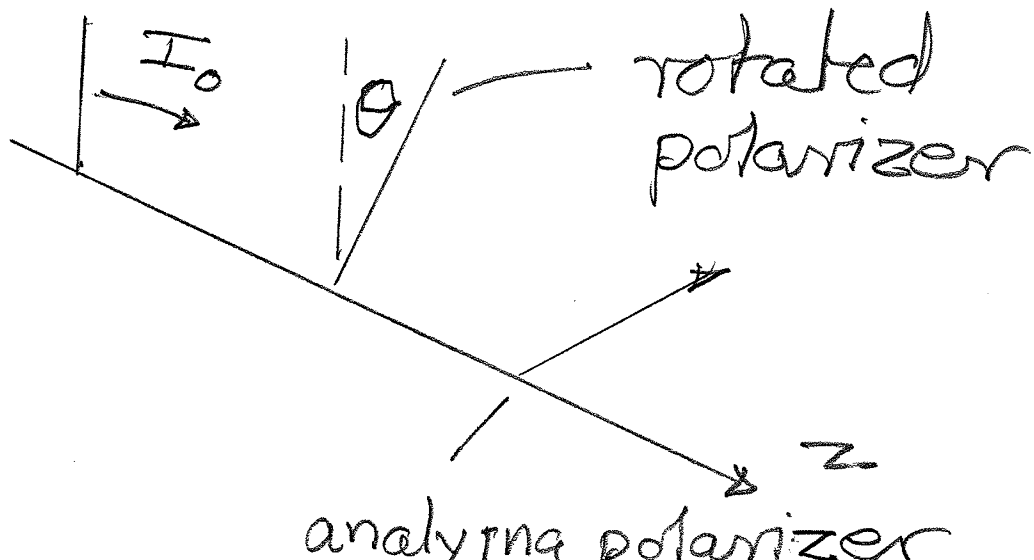
$$(A_y/A_x) = 1.$$

This produce elliptically polarized light whose handedness can be changed using $\Delta\phi$

Eg, $\Delta\phi = -\pi$,

$$\boxed{LHC \longrightarrow RHC}$$

• As a final example we consider a pair of crossed polarizers with a third rotated polarizer between them.



See the Applet on webpage.

Take input polarized along x

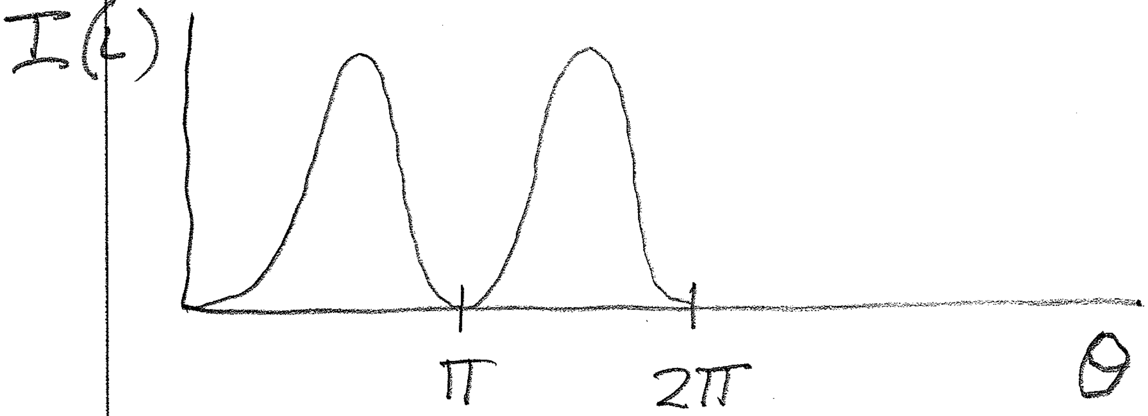
$$\bar{\mathcal{E}}(L) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{E}_0 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\mathcal{E}_0 \sin(2\theta) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

which is polarized along y

This yield the output intensity

$$I(L) = \frac{I_0}{4}\sin^2(2\theta).$$



So we can get light transmitted even though there are crossed polarizers involved!! This is a strictly wave optical phenomenon that further relies on the vector nature of light.

Spin angular momentum

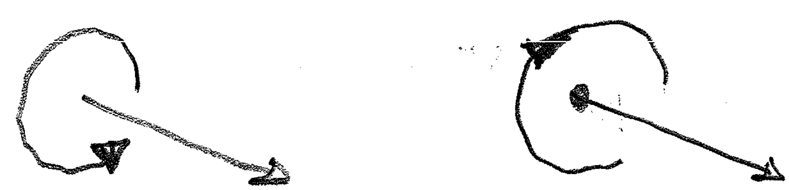
We have seen before that light carries energy and linear momentum

$$E = \hbar\omega \quad \text{per photon}$$

$$\vec{p} = \hbar\vec{k} \quad \text{per photon}$$

Circularly polarized light also carries spin angular momentum associated with the rotating electric field

$$\vec{S} = \pm\hbar\hat{k} \quad \text{per photon}$$



Linear polarization, which is an equal combination of LHC & RHC carries no spin angular momentum.

Optical spanner — see optical rotation movies