Optical fields & Laser beams

Maxwell's equation describe EM waves over the full spectrum - see Fig. The visible spectrum covers the range.

$\lambda = 400 - 700\,nm$

$\nu = c/\lambda = 4.3\times10^{14} - 7.5\times10^{14}\,Hz.$

$= \omega/2\pi.$ (ROYGBIV)

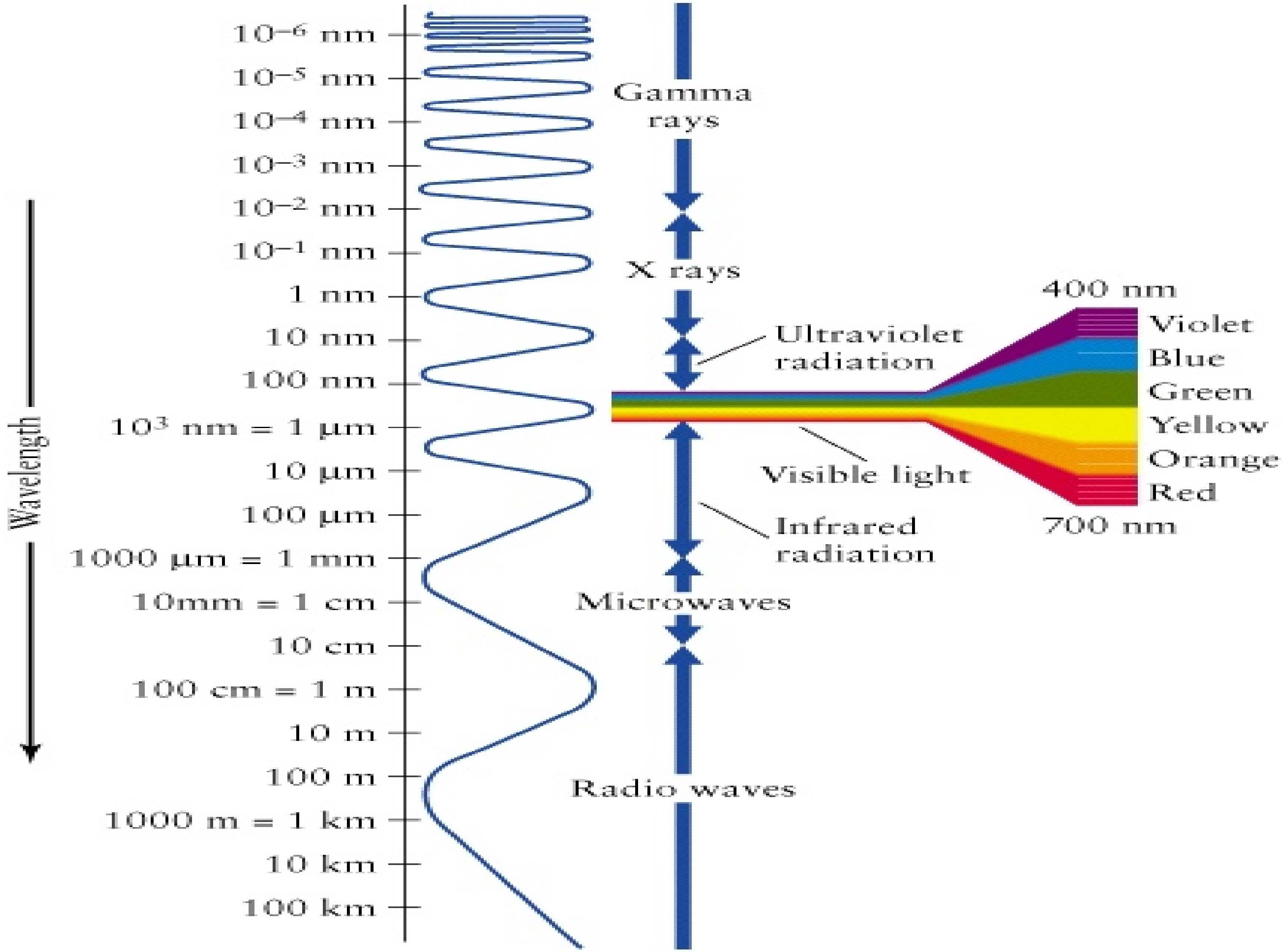
You should be conversant with these ranges and the basic relations

$\omega = ck = 2\pi\nu$, $k = (2\pi/\lambda) = 2\pi\bar{\nu}$,

$\bar{\nu} = 1/\lambda$, $\nu = c/\lambda$, & terminology:

Lasers are sources of near monochromatic EM radiation, they have well defined λ or ν. They have narrow bandwidth $\Delta\lambda/\lambda \lll 1$.

Electromagnetic Spectrum



Laser Properties

High spectral purity & intensity…

Remain collimated:
Lunar range finder…

Non-laser sources, eg. light bulbs are not narrow bandwidth $\Delta\lambda/\lambda < 1$.

The output of lasers can be approximated as harmonic waves with well defined polarization $\hat{e}$

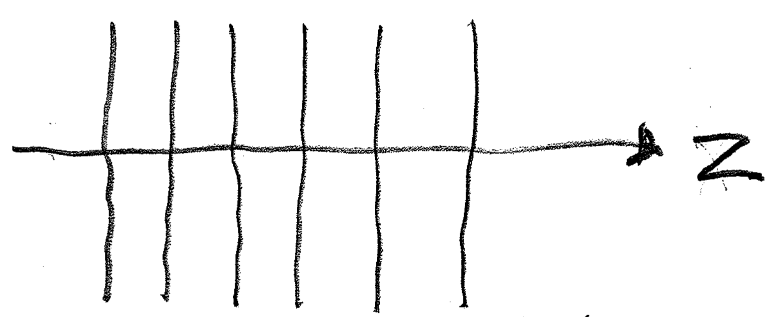
$$\vec{E}(\vec{r},t) = \hat{e}\, E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varepsilon)$$

Lasers are phase coherent — ε constant.

Take an EM field propagating along the z-axis through free-space

$$\vec{E}(\vec{r},t) = \hat{\imath}\, E_0 \cos(kz - \omega t),$$

$\varepsilon = 0$, $\omega = ck$. This is a plane-wave electric field,



CW plane-wave field.

and the time-averaged Poynting vector is

$$\langle \vec{S} \rangle_T = \langle \vec{E} \times \vec{B} \rangle_T / \mu_0$$
$$= \hat{k} I = \hat{k} \frac{1}{2} \epsilon_0 n c E_0^2$$

Consider example of HeNe laser beam ($\lambda = 632$ nm) in free-space ($n=1$), and $I \sim 1$ MW/cm^2.

$$I = 10^6 \times 10^4 \text{ W/m}^2 \text{ (SI)}.$$

($I \sim 10^3$ W/m^2 on Earth due to sun).

then

$$E_0 = \sqrt{\frac{2I}{\epsilon_0 c}} = 2.7 \times 10^6 \text{ V/m}$$

Electric field (Coulomb) binding electron to nucleus $\sim 10^9$ V/m !

Increase E_0 by 300, I by $(300)^2$

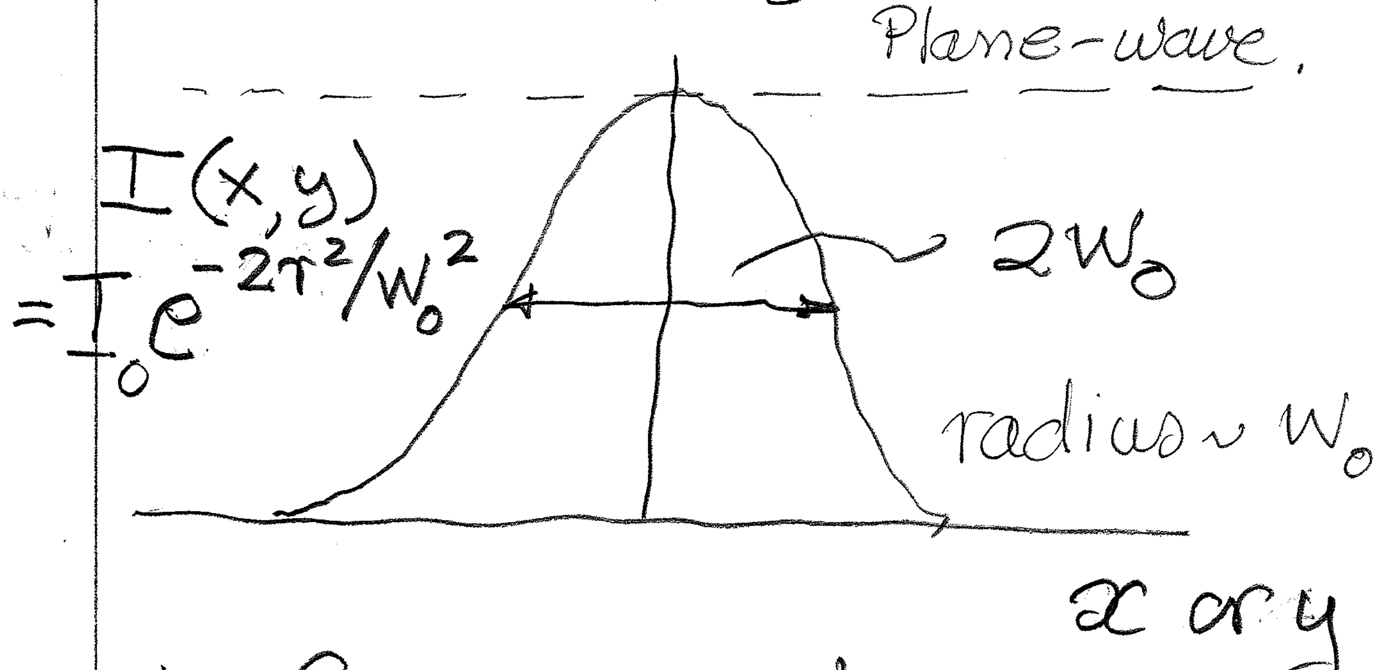
$I \sim 10^{11}$ W/cm^2

The power of a laser beam (in W) is the intensity integrated over the beam cross-sectional area A

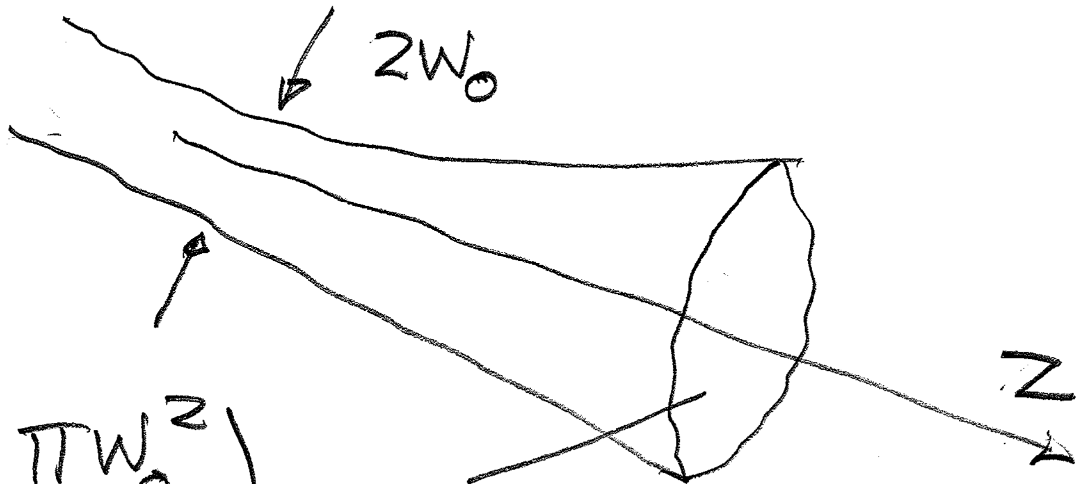
$$P = \iint_A dS \, I \, (W/m^2)$$

dS — area m^2

power in W. Plane-waves have infinite cross sectional area, so infinite power! Not physical. Real laser beams have finite cross sections, eg. Gaussian



w_0 - Gaussian spot size.



$$A \sim \left(\frac{\pi W_0^2}{2}\right)$$

Beam cross-section.

The beam power is then

peak intensity.

$$P \sim \left(\frac{\pi W_0^2}{2}\right) \cdot I \quad (\text{in W}).$$

Consider HeNe example with

$I \sim 1 \text{MW/cm}^2$, $W_0 = 10\,\mu m$ ($\gg \lambda$),

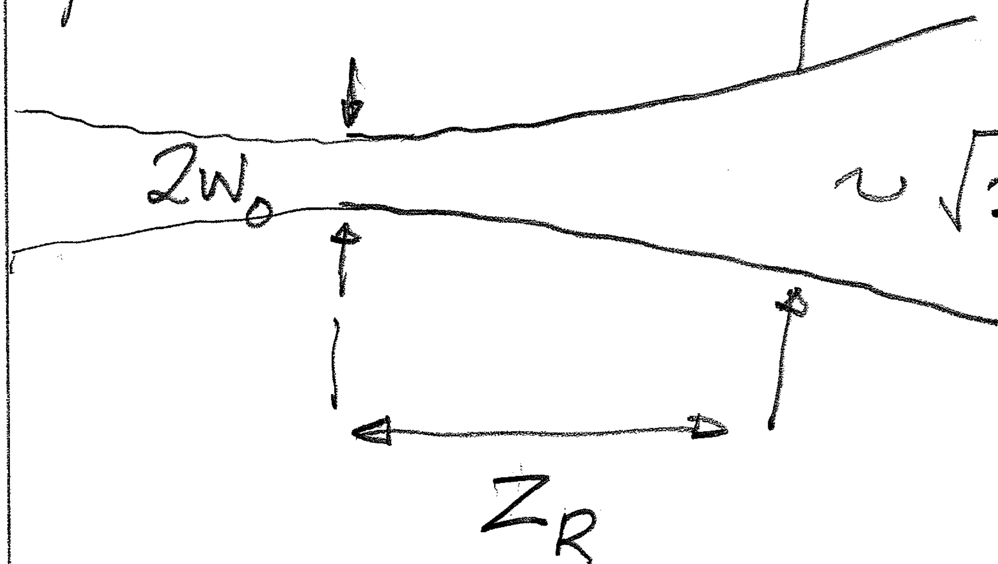
then $\qquad = 10^{-3}$ cm

$$P \sim \frac{\pi}{2} \times (10^{-3})^2 \times 10^6$$

$$\sim 1.6\,W.$$

(typically ~ 10mW - 1W for HeNe)

The price for having a finite cross-section is that the laser beam spreads via diffraction (Opti 330)



L_R is the Rayleigh range - distance over which the laser beam stays approx constant, collimated

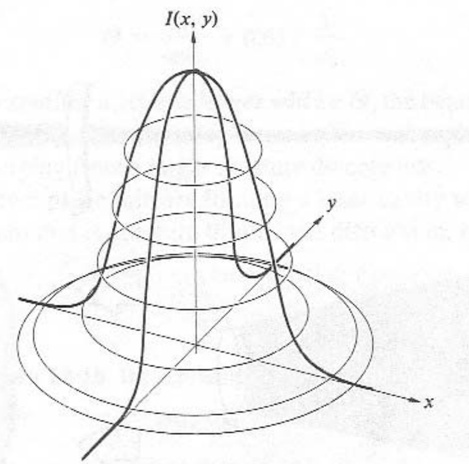
$$\boxed{Z_R \sim \frac{\pi W_0^2}{\lambda}}$$ (Good if $W_0 \gg \lambda$)

$W_0 = 100\mu m$, $\lambda = 0.632\mu m$, $Z_R \sim 5cm$

$W_0 = 1cm$, $\lambda = 0.632\mu m$, $Z_R \sim 500m$

$W_0 = 0.1m$, " ", $Z_R \sim 5 \times 10^3 Km$!

Gaussian beams



- Zero order mode is Gaussian
- Intensity profile: $I = I_0 e^{-2r^2/w^2}$
- beam waist: w_0

$$w = w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2}$$

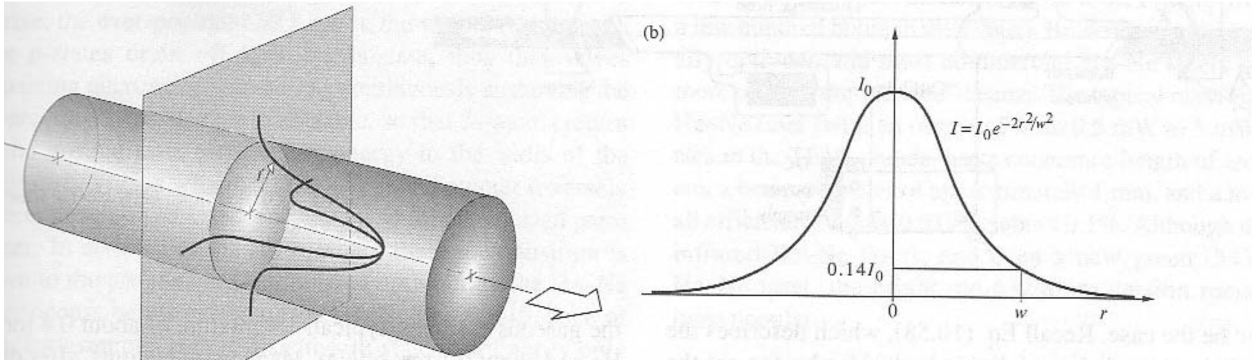


Figure 13.14 A Gaussian beam-like wave propagating in the z-direction.

- confocal parameter:

$$z_R = \frac{\pi w_0^2}{\lambda}$$

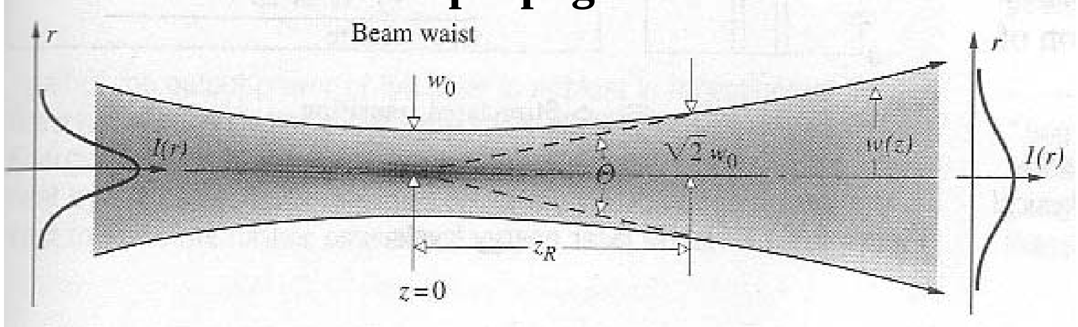
- far from waist

$$w \to \frac{\lambda z}{\pi w_0}$$

- divergence angle

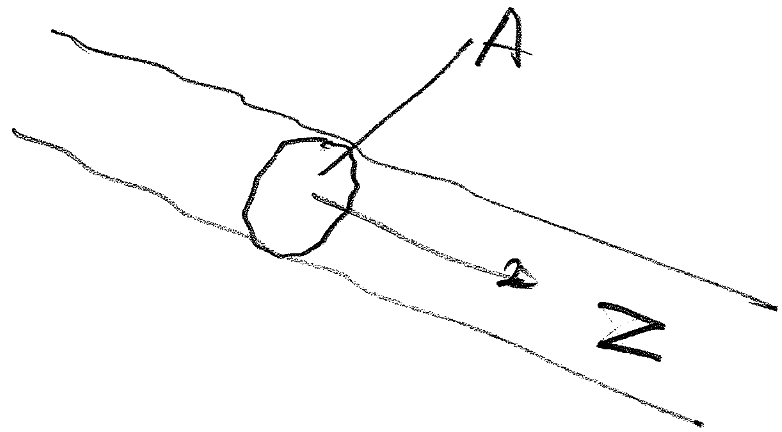
$$\Theta = \frac{2\lambda}{\pi w_0} = 0.637 \frac{\lambda}{w_0}$$

Gaussian propagation



Photon considerations (Hecht 3.3.3–3.3.4)

The beam power P is in W or $J s^{-1}$, energy/time. This is the energy/time crossing the beam cross section



Now each photon has energy $\hbar\omega$, so the # of photons/time, the photon flux Φ, crossing the area is

$$\Phi = \frac{P}{\hbar\omega} = \frac{\left(\frac{\pi w_0^2}{2}\right) I}{\hbar\omega}.$$

Take a beam with $P = 1W$ @ $\lambda = 0.632\mu m$

so $\omega = kc = 2\pi c/\lambda = 3\times 10^{15}\ rad\ s^{-1}$.

Then using $\hbar = 1.0\times 10^{-34}\ Js^{-1}$,

$$\Phi \simeq 1\times 10^{17}\ \text{photons/sec.}$$

Typical lab lasers have lots of photons!

Laser fields also carry momentum, $\hbar k$/photon in the laser direction.

Now

$$\Phi\cdot(\hbar k) = \frac{\text{photons}}{\text{time}}\cdot \hbar k.$$

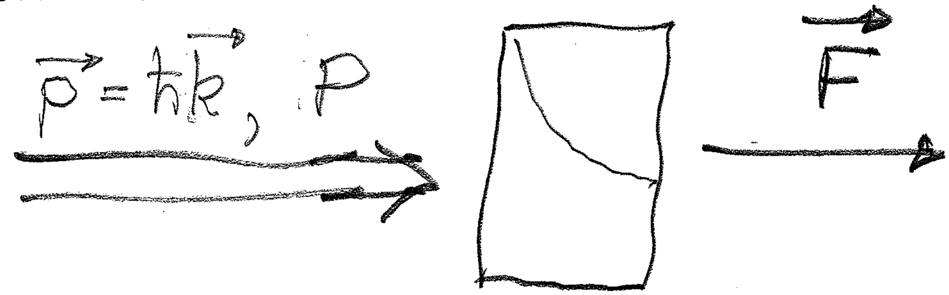
and force $\equiv$ rate of change of momentum.

so

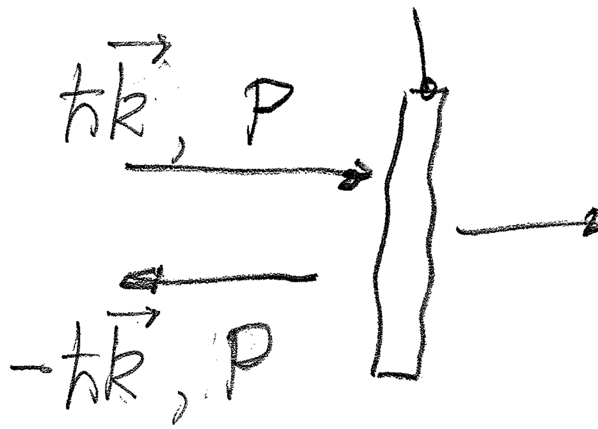
$$\vec{F} = \hat{k}\,\Phi\cdot(\hbar k) = \hat{k}\frac{P}{\hbar\omega}\cdot\hbar k$$

$$= \hat{k}\frac{P}{c}$$

P/C is the force that would be exerted on a body that absorbed the laser



or 2P/c for a reflected beam



$$\vec{F} = \hat{\imath}\left(\frac{2P}{c}\right).$$

This is called radiation pressure and has been measured for light mirrors that are suspended

$\longrightarrow$ Light carries energy

& momentum.

Optical interference I.

Here we shall look at the interference between two optical fields described by harmonic waves – real field description

$$\vec{E}(\vec{r},t) = \hat{e}_1 E_1 \cos(\vec{k}_1\cdot\vec{r} - \omega_1 t + \varepsilon_1) + \hat{e}_2 E_2 \cos(\vec{k}_2\cdot\vec{r} - \omega_2 t + \varepsilon_2)$$

The magnitude of the Poynting vector is (no time average yet)

$$\frac{S(\vec{r},t)}{\epsilon_0 n c} = \vec{E}(\vec{r},t)\cdot\vec{E}(\vec{r},t)$$

$$= E_1^2 \cos^2(\vec{k}_1\cdot\vec{r} - \omega_1 t + \varepsilon_1) \qquad S_1$$

$$+ E_2^2 \cos^2(\vec{k}_2\cdot\vec{r} - \omega_2 t + \varepsilon_2) \qquad S_2$$

$$+ 2(\hat{e}_1\cdot\hat{e}_2) E_1 E_2 \cos(\vec{k}_1\cdot\vec{r} - \omega_1 t + \varepsilon_1)$$

$$S_{12} \qquad \times \cos(\vec{k}_2\cdot\vec{r} - \omega_2 t + \varepsilon_2)$$

S_1 & S_2 are magnitudes of Poynting vectors of individual beams, S_{12} is the interference term, cross term.

If the two fields have orthogonal polarizations $(\hat{e}_1 \cdot \hat{e}_2) = 0$, and there is no interference

We take the case $\hat{e}_1 = \hat{e}_2$, $(\hat{e}_1 \cdot \hat{e}_2) = 1$, and $\varepsilon_1 = \varepsilon_2 = 0$

$$\frac{S(\vec{r},t)}{\epsilon_0 n c} = \underbrace{E_1^2 \cos^2(\vec{k}_1 \cdot \vec{r} - \omega_1 t)}_{S_1} + \underbrace{E_2^2 \cos^2(\vec{k}_1 \cdot \vec{r} - \omega_1 t)}_{S_2} + \underbrace{2 E_1 E_2 \cos(\vec{k}_1 \cdot \vec{r} - \omega_1 t) \cos(\vec{k}_2 \cdot \vec{r} - \omega_2 t)}_{S_{12}}$$

Now

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

so the interference term becomes.

$$\cos(\vec{k}_1\cdot\vec{r}-\omega_1 t)\cos(\vec{k}_2\cdot\vec{r}-\omega_2 t)$$

$$=\frac{1}{2}\Big[\cos((\vec{k}_1+\vec{k}_2)\cdot\vec{r}-(\omega_1+\omega_2)t)$$ fast oscillating

$$+\cos((\vec{k}_1-\vec{k}_2)\cdot\vec{r}-(\omega_1-\omega_2)t)\Big]$$

slow oscillating if $\omega_1 \simeq \omega_2$

On time averaging we lose the first term. If we time average S_1 or S_2

$$\frac{\langle S_1\rangle_T}{\epsilon_0 n c}=\frac{1}{2}E_1^2,\quad \frac{\langle S_2\rangle_T}{\epsilon_0 n c}=\frac{1}{2}E_2^2$$

$$\frac{\langle S_{12}\rangle_T}{\epsilon_0 n c}=E_1E_2\cos(\overrightarrow{\Delta k}\cdot\vec{r}-\Delta\omega t)$$

$$\overrightarrow{\Delta k}=(\vec{k}_1-\vec{k}_2),\quad \Delta\omega=\omega_1-\omega_2.$$

and finally

$$\langle S \rangle_T = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos(\vec{\Delta k}\cdot\vec{r} - \Delta\omega t)$$

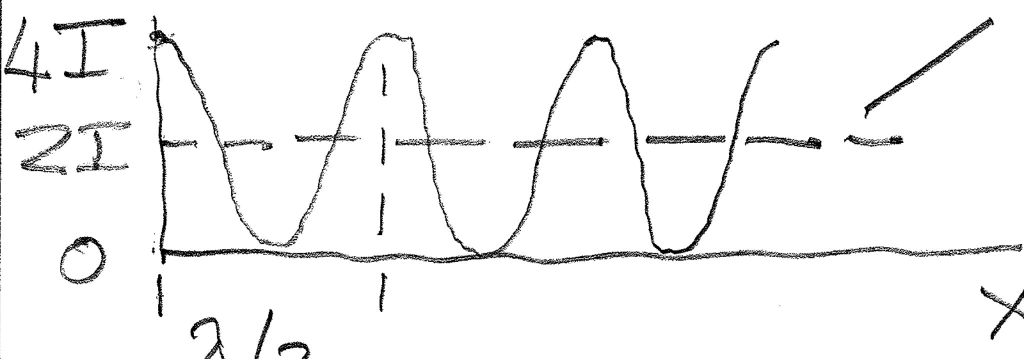
$$\vec{\Delta k} = (\vec{k}_1 - \vec{k}_2),\ \Delta\omega = \omega_1 - \omega_2 .$$

Ex. $\vec{k}_1 = k\hat{i},\ \vec{k}_2 = -k\hat{i},\ \omega_1 = \omega_2$

$$\xrightarrow{\quad 1 \quad}\ \xleftarrow{\quad 2 \quad}$$

$I_1 = I_2 = I$, two identical counterpropagating beams ($\vec{\Delta k} = 2k\hat{i}$, $\Delta\omega = 0$)

$$\langle S \rangle_T = 2I(1 + \cos(2kx - \Delta\omega t))$$

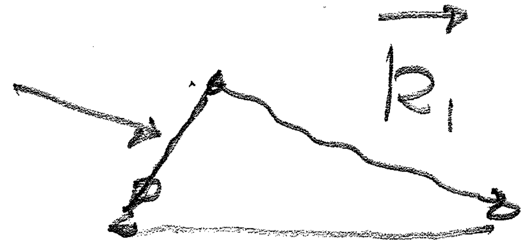


result for $(\hat{e}_1 \cdot \hat{e}_2) = 0$

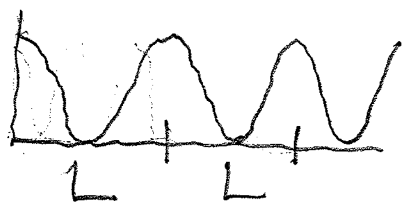
Standing wave interference pattern.
(Moves if $\Delta\omega \neq 0$).

Ex.

$(\vec{k}_1 - \vec{k}_2)$



Leads to time average interference pattern ($\Delta\omega = 0$)

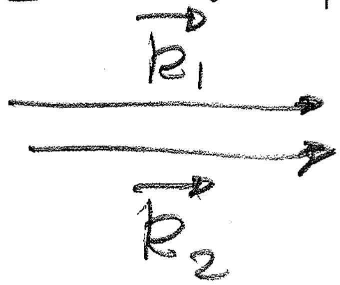


Spacing of the fringes L

$$|\vec{k}_1 - \vec{k}_2| L = \Delta k L = 2\pi$$

$$L = 2\pi / \Delta k$$

Next we look at the field itself for the case $\vec{k}_1 = k_1\hat{i}$, $\vec{k}_2 = k_2\hat{i}$, $\varepsilon_1 = \varepsilon_2 = 0$, $\hat{e}_1 = \hat{e}_2 = \hat{e}$, $E_1 = E_2 = E_0$



Co-propagating
same pol'z'n.

$$\vec{E}(\vec{r},t) = \hat{e}E_0\left[\cos(k_1x - \omega_1t) + \cos(k_2x - \omega_2t)\right]$$

$$= 2\hat{e}E_0\cos\left[\left(\frac{k_1+k_2}{2}\right)x - \frac{(\omega_1+\omega_2)}{2}t\right]$$

$$\times\cos\left[\frac{(k_2-k_1)}{2}x - \frac{(\omega_2-\omega_1)}{2}t\right]$$

Define

$$\bar{k} = \frac{(k_1+k_2)}{2}, \quad \bar{\omega} = \frac{(\omega_1+\omega_2)}{2}$$

$$\Delta k = \left(\frac{k_2-k_1}{2}\right), \quad \Delta\omega = (\omega_2-\omega_1)/2$$

$$\vec{E}(\vec{r},t) = 2\hat{e}E_0\cos(\bar{k}x - \bar{\omega}t)\cos(\Delta kx - \Delta\omega t)$$

If $k_1 \simeq k_2$ and $\omega_1 \simeq \omega_2$, then

$\cos(\bar{k}x - \bar{\omega}t)$ oscillates fast - carrier
$\cos(\Delta k x - \Delta\omega t)$ oscillates slow envelope
Use Matlab code to show carrier fringes vs. envelope.
With time dependence both move

Carrier

$$\cos(\bar{k}x - \bar{\omega}t) \longrightarrow \cos\left(\bar{k}\left(x - \left(\frac{\bar{\omega}}{\bar{k}}\right)t\right)\right)$$

giving the fringe velocity or phase velocity

$$\frac{c}{n} = V_p = \frac{\bar{\omega}}{\bar{k}} = \frac{\omega}{k} \quad \left(\text{as } \omega_1 = \omega_2 = \omega, \; k_1 = k_2 = k\right)$$

Envelope

$$\cos(\Delta k x - \Delta\omega t) \rightarrow \cos\left(\Delta k\left(x - \left(\frac{\Delta\omega}{\Delta k}\right)t\right)\right)$$

giving the group velocity

$$V_g = \frac{\Delta \omega}{\Delta k} \longrightarrow \frac{\partial \omega}{\partial k}.$$

These two velocities can be different!

$$V_g = \frac{\partial \omega}{\partial k} = \left(\frac{\partial k}{\partial \omega}\right)^{-1}$$

Now $k = \omega / V_p = n\omega / c$, so

$$\frac{\partial k}{\partial \omega} = \frac{\partial}{\partial \omega}\left(\frac{n\omega}{c}\right)$$

$$= \frac{1}{c}\left(n + \omega \frac{\partial n}{\partial \omega}\right)$$

$$= \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{\partial n}{\partial \omega}\right)$$

$$= \frac{1}{V_p}\left(1 + \frac{\omega}{n}\frac{\partial n}{\partial \omega}\right) = \frac{1}{V_g}$$

and

$$V_g = V_p \frac{1}{\left(1 + \frac{\omega}{n}\frac{\partial n}{\partial \omega}\right)}$$

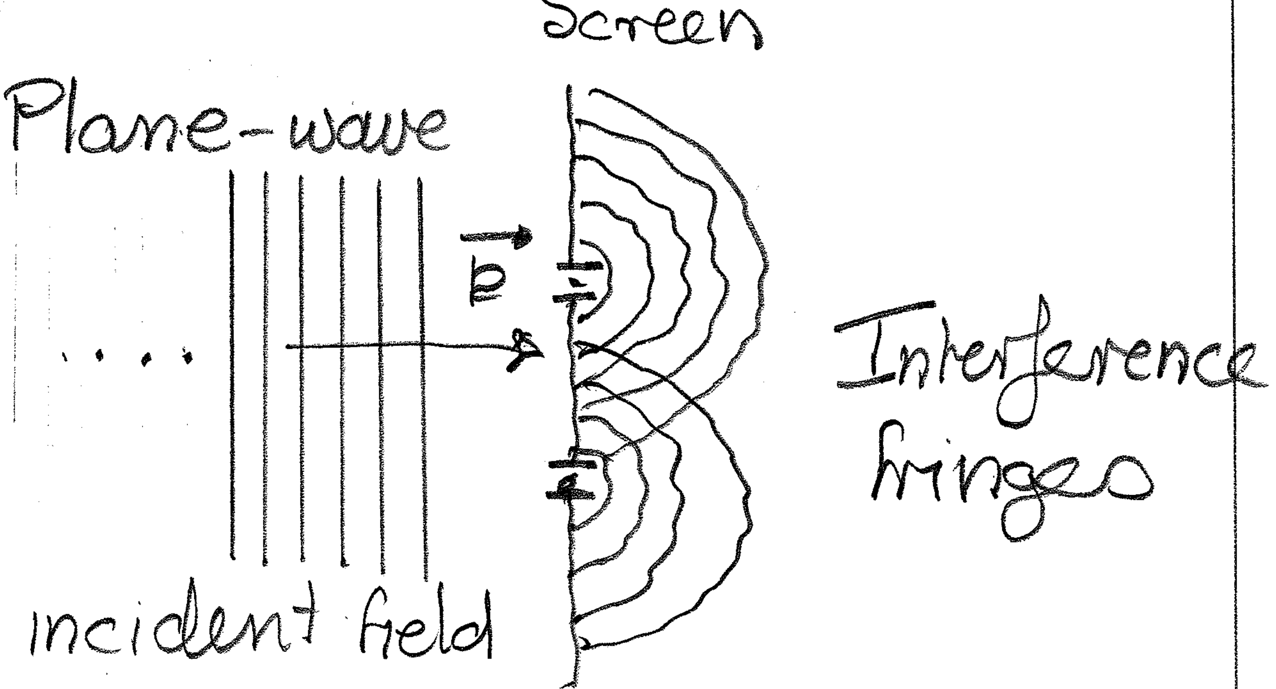
If the medium refractive-index varies with ω or λ (called optical dispersion) V_p & V_g can be (very) different

Carrier fringes move @ V_p
envelope " " @ V_g.

Use Matlab code to illustrate,
If $V_p = c/n$ depends on ω or λ
then V_g and V_p can be different.

Optical interference II

Here we consider Young's two-slit experiment (see Fig.)

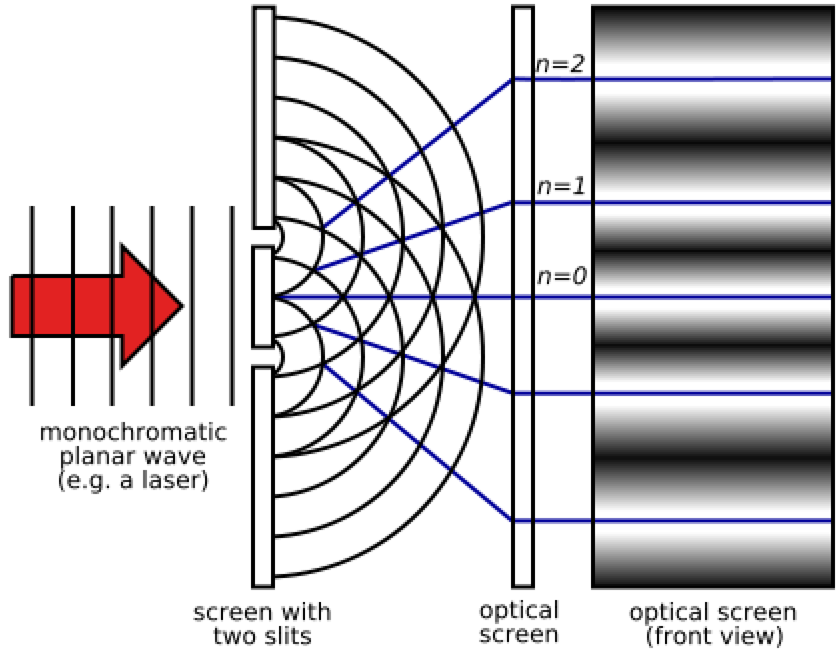


Two slits act as sources of spherical waves - analogy with interference of water waves - see webpage.

Example of division of wavefront.
- takes a wavefront, divides it in two, and then recombines to produce interference.

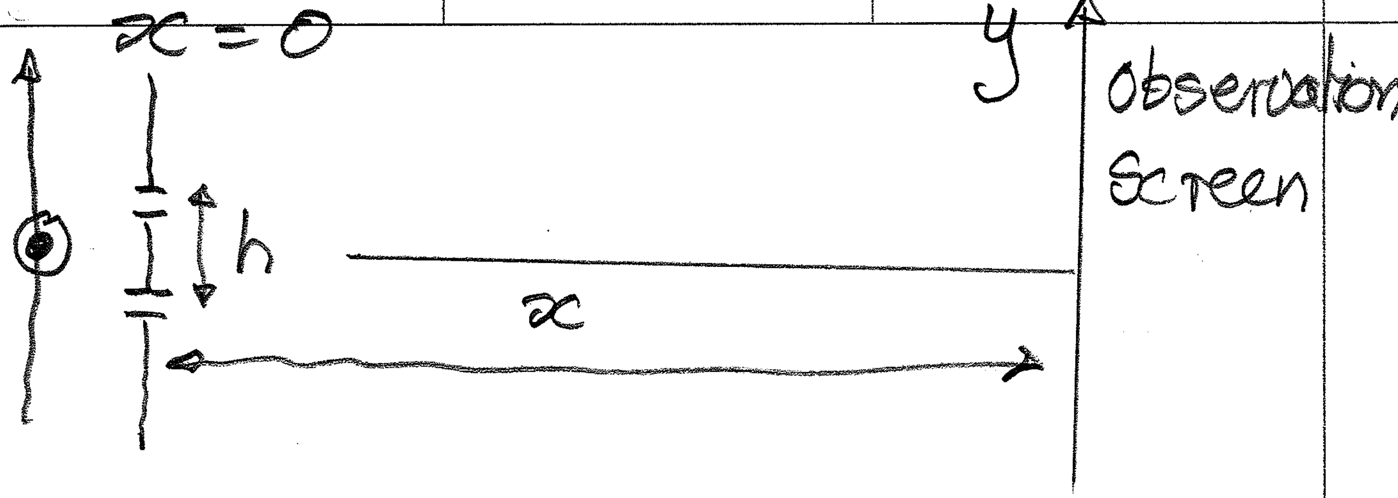
Young's two-slit experiment

- Paradigm of optical interference
- Displays wave nature of EM field
- Path difference argument
- Simple with lasers
- Single photons?



Water wave interference

- Simple physical system
- Fringes far from source
- Far field pattern



Field polarized along the z-axis.
At a point y on the observation screen.

$$\vec{E}(\vec{r},t) = \frac{\mathcal{E}\,\hat{k}}{|\vec{r}-\hat{j}h/2|}\, e^{i(k|\vec{r}-\hat{j}h/2|-\omega t+\varepsilon)}$$

$$+\ \frac{\mathcal{E}\,\hat{k}}{|\vec{r}+\hat{j}h/2|}\, e^{i(k|\vec{r}+\hat{j}h/2|-\omega t+\varepsilon)}.$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Consider $z=0$,

$$|\vec{r} \pm \hat{j}h/2| = |x\hat{i} + (y \pm h/2)\hat{j}|$$

$$= \sqrt{x^2 + (y \pm h/2)^2}$$

Assume $x >> |y|, h$ (far field).

$$|\vec{r} \pm \hat{j} h/2| = x \sqrt{1 + \frac{(y \pm h/2)^2}{x^2}}$$

(Taylor expand)

$$\simeq x \left(1 + \frac{(y \pm h/2)^2}{2x^2}\right)$$

$$\simeq x \left(1 + \frac{1}{2}\left(\frac{y}{x}\right)^2 \pm \frac{yh}{2x^2} + \frac{1}{8}\left(\frac{h}{x}\right)^2\right)$$

$$\simeq \underbrace{L}_{\text{large}} \pm \underbrace{\frac{yh}{2x}}_{\text{small correction}} = |\vec{r} \pm \hat{j} h/2|$$

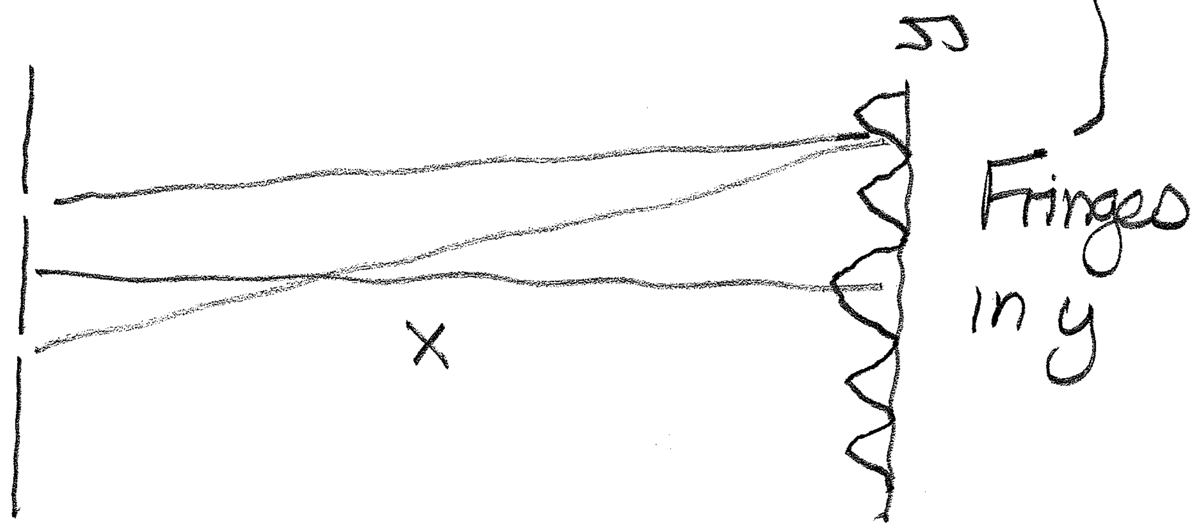
$$\vec{E}(\vec{r}, t) \sim \hat{k} \frac{\mathcal{R}}{x} e^{i(kL + kyh/2x - \omega t + \varepsilon)}$$

$$+ \hat{k} \frac{\mathcal{R}}{x} e^{i(kL - kyh/2x - \omega t + \varepsilon)}$$

$$= \hat{k} \frac{2\mathcal{R}}{x} e^{i(kL - \omega t + \varepsilon)} \cos\left(\frac{kyh}{2x}\right)$$

So the physical field is

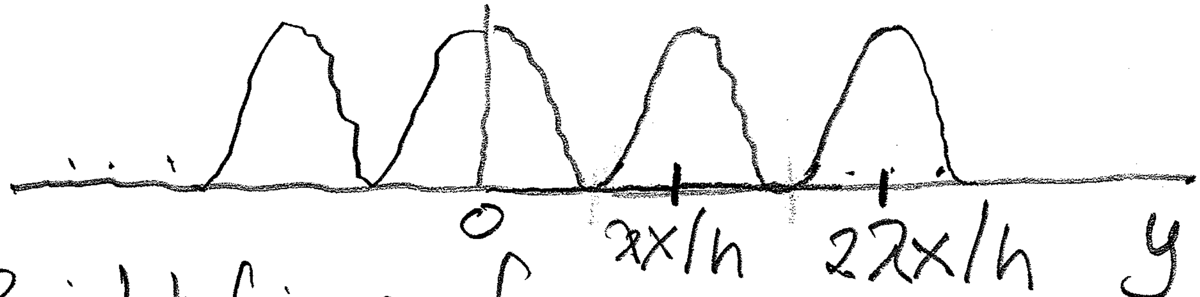
$$\vec{E}(\vec{r},t) = \hat{R}\left(\frac{2A}{x}\right)\cos(kL-\omega t+\varepsilon)$$
$$\times\cos\left(\frac{kyh}{2x}\right)$$



We can calculate the magnitude of the time averaged Poynting vector

$$\frac{\langle S\rangle_T}{\varepsilon_0 nc} = \langle\vec{E}\cdot\vec{E}\rangle_T$$
$$= \left(\frac{2A}{x}\right)^2\cdot\frac{1}{2}\cdot\cos^2\left(\frac{kyh}{2x}\right)$$

$$\frac{kyh}{2x} = \frac{2\pi}{\lambda}\cdot\frac{yh}{2x} = \pi\cdot\left(\frac{yh}{\lambda x}\right)$$



Bright fringes for

$$\frac{kyh}{2X} = m\pi, \ m = 0, \pm 1, \pm 2 \ldots$$

or

$$y = m \cdot \left(\frac{\lambda x}{h}\right) = 0, \pm\left(\frac{\lambda x}{h}\right) \ldots$$

The bright fringe spacing is

$$\Delta y = \frac{\lambda x}{h}.$$

Lesson: Adding harmonic waves leads to optical interference which is a the heart of much optics – interferometers, metrology....