4. Review of Electromagnetism (Hecht 3.1).

The material in this section should be familiar from Phys. 241, though there may be notational differences with the discussion in Hecht 3.1. You should review the appropriate material as it appears. (SI units throughout)

Our discussion stems from key experimental observations underpinning EM theory:

1. Electric & magnetic fields exert forces on charged particles (Lorentz)
2. A time-varying magnetic field has an electric field associated with it (Faraday)
3. Flux of EM field is zero in a charge free region (Gauss).
4. A time-varying electric field has an associated magnetic field (Ampere)

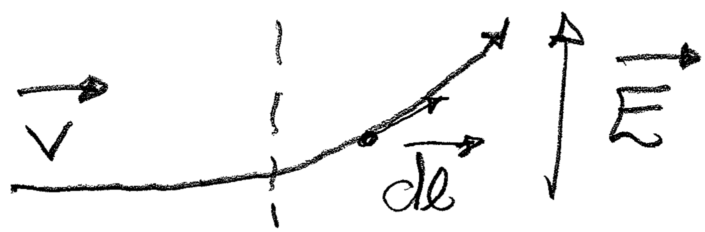
Lorentz force law

A charged particle of charge q and velocity $\vec{v}$ moving through electric & magnetic fields $\vec{E}$ & $\vec{B}$ experiences a force given by the Lorentz law.

$$\vec{F} = \vec{F}_E + \vec{F}_M = q\vec{E} + q(\vec{V} \times \vec{B})$$

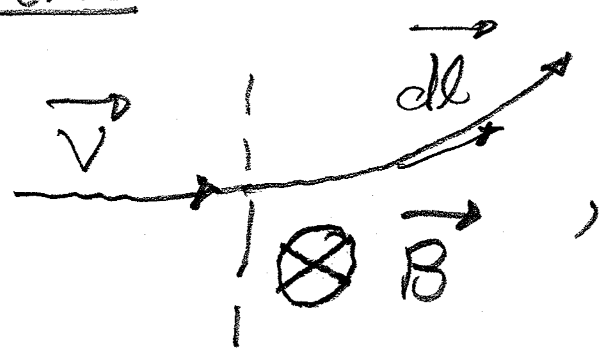
$\vec{E}$ & $\vec{B}$ – vector EM fields. eg. $\vec{E}(\vec{r}, t)$

Eg. Static $\vec{E}$ field $\vec{B} = 0$



$\vec{F}_E \parallel \vec{E}$

or static $\vec{B}$ field $\vec{E} = 0$



$\vec{F}_M \perp \vec{V}$ & $\vec{B}$

charged particle deflection by EM field.

Consider the work done as the particle moves a displacement $\vec{dl} \parallel \vec{v}$,

$$dW = \vec{F} \cdot \vec{dl} = \vec{F_E} \cdot \vec{dl} + \vec{F_M} \cdot \vec{dl}$$

$$= q\,\vec{E} \cdot \vec{dl} + q\,\vec{dl} \cdot (\vec{v} \times \vec{B})$$

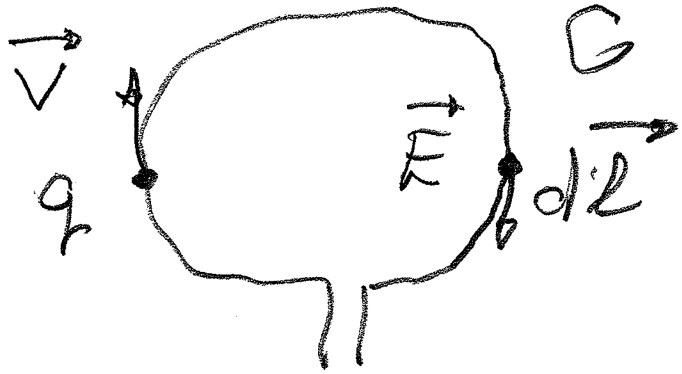
Now $\vec{dl} \parallel$ to $\vec{v}$, so $\vec{dl} \cdot \vec{F_M} = 0$, the magnetic term does no work on the charged particle. Then the total work done on a curve C in space is

$$W = q \int_C \vec{E} \cdot \vec{dl} = \text{Work done on charge}$$

For a <u>closed</u> loop the work done is $W = q \times \text{emf}$, <u>E</u>lectro<u>m</u>otive <u>f</u>orce – bad name, it is a voltage!

$$W = q \oint_C \vec{E} \cdot \vec{dl} = q \times \text{emf}$$

Eg. a wire loop



Voltage diff or emf

charges driven around wire loop C by field $\vec{E}$ on the loop

$$\text{emf} = \text{Voltage diff} = \oint_C \vec{E} \cdot \vec{dl}$$

The emf is an experimentally observable quantity - if $\vec{E} = 0$, and a static $\vec{B}$ field is present then emf $= 0$

How about $\vec{B}$ time varying with $\vec{E} = 0$? Not possible!

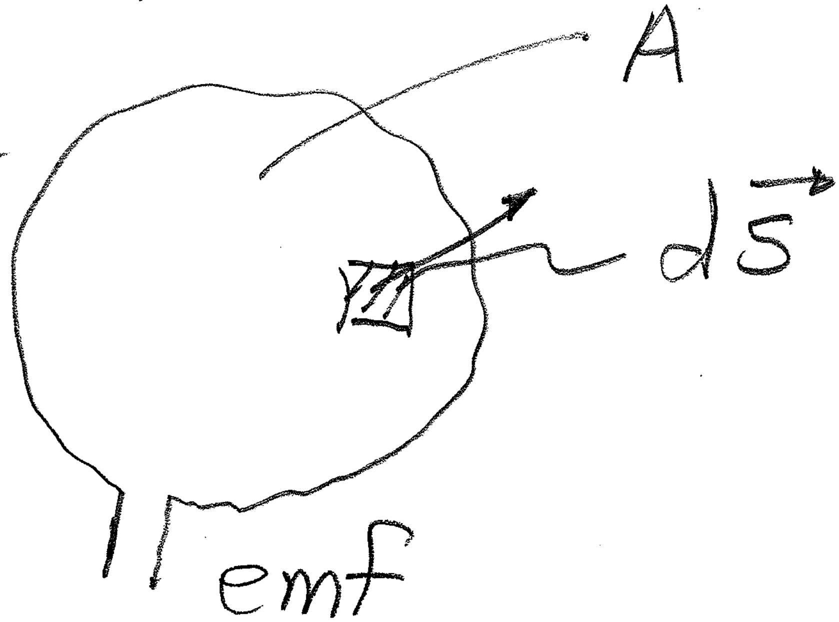
Faraday's law

It is experimentally known that if a time-dependent magnetic field pierces the wire loop an emf is also generated

$$\text{emf} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt}\iint_A \vec{B}\cdot d\vec{S}$$

with $\Phi_M = \iint_A \vec{B}\cdot d\vec{S}$ the magnetic flux.

A is the wire loop area



Note, if $\vec{B}$ is time-independent

$\Phi_M = \text{const}$, $\text{emf} = 0$.

If $\vec{B}$ is time-varying an emf can result, but since

$$\text{emf} = \oint_C \vec{E} \cdot d\vec{l}$$

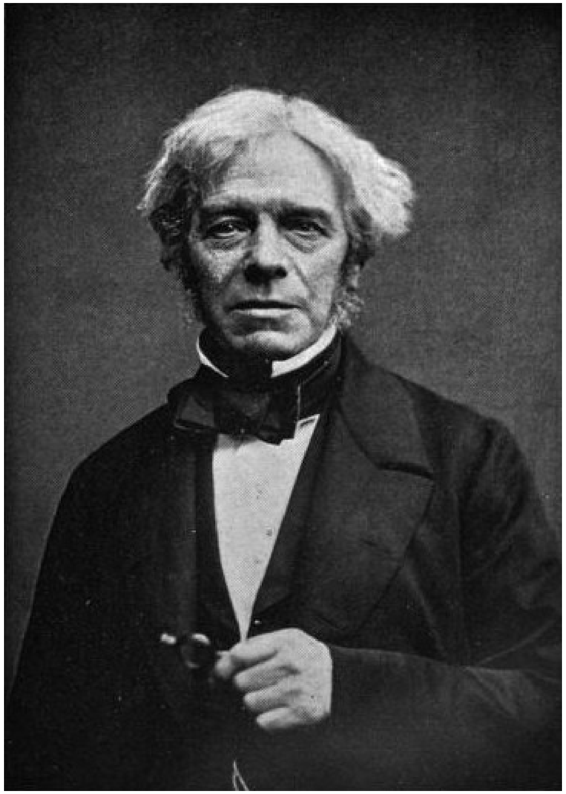
an associated electric field must be present! Comparing the two forms of emf we get <u>Faraday's law</u>

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{S}$$

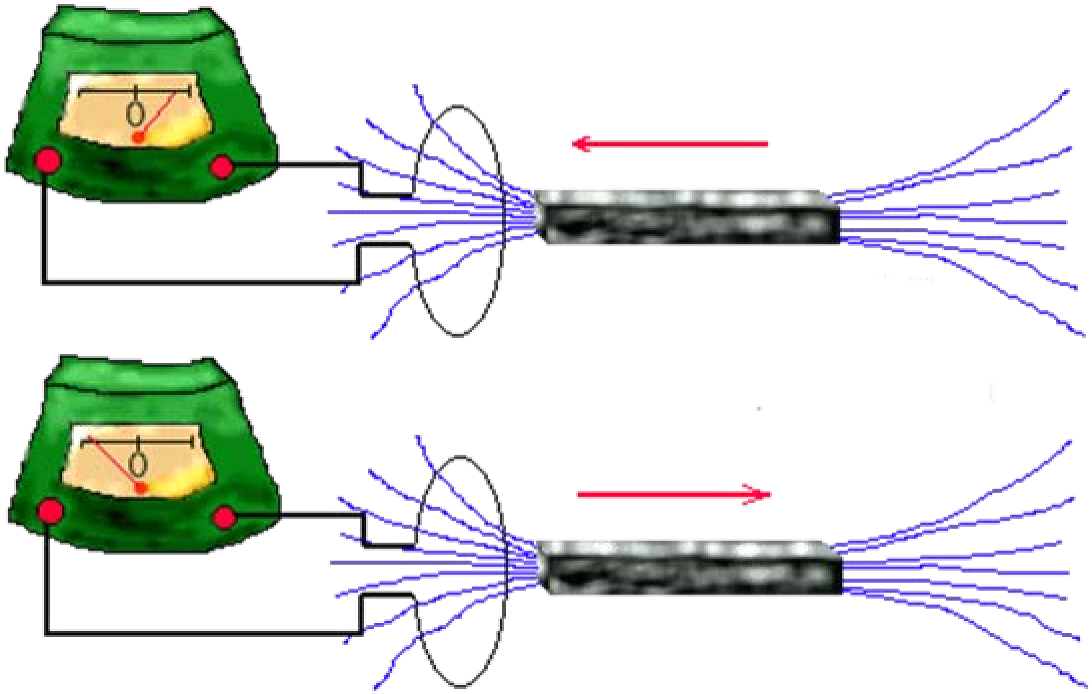
See the next page for the expt. arrangement to test Faraday's law for time-varying magnetic fields.

Faraday's law

Michael Faraday



EMF from a time varying magnetic field



Gauss' Law

The ultimate source of EM fields is the motion of charged particles

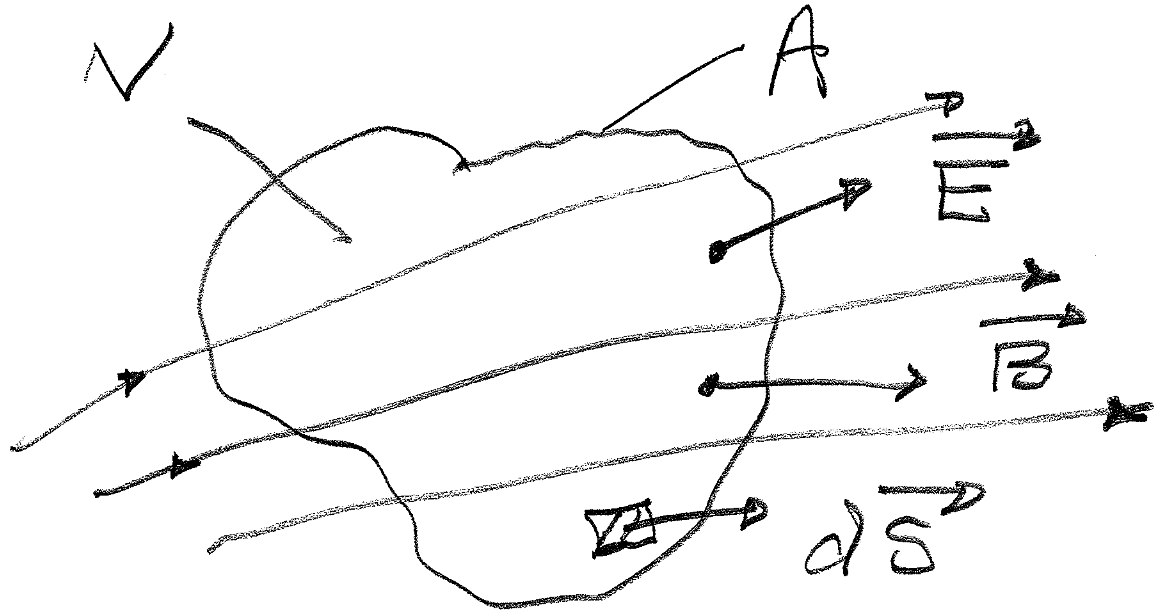
- Laser - electrons in atoms
- Antenna - electrons in wires
- Cyclotron - accelarating charges

In a volume with no charges there is no source for the fields and it follows that the flux of the EM fields will be zero

$$\Phi_E = \iiint_V dV\, \nabla \cdot \vec{E} = \oint\!\!\!\oint_A d\vec{S} \cdot \vec{E} = 0$$

$$\Phi_M = \iiint_V dV\, \nabla \cdot \vec{B} = \oint\!\!\!\oint_A d\vec{S} \cdot \vec{B} = 0$$

where Φ_E & Φ_B are the fluxes, and Gauss' theorem was used



net flux = 0.

Field lines in ≡ Field lines out.

To date there is no evidence for magnetic charges (monopoles), and Gauss' law for the magnetic field is

$$\boxed{\Phi_M = \oiint_A d\vec{S} \cdot \vec{B} = 0}$$

If the volume contains a total charge $\sum q$, then the electric flux is

$$\Phi_E = \oiint_A d\vec{S} \cdot \vec{E} = \frac{1}{\epsilon_0} \sum q$$

This is Gauss' law for the electric field, where $\epsilon_0 = 8{,}85 \times 10^{-12}\ C^2/N \cdot m^2$ is the permittivity of free-space.

Dielectric constant

Above we have tacitly assumed that the charges are embedded in vacuum - what if the charges are in, eg. a gas or liquid? We replace $\epsilon_0 \to K_E \epsilon_0$ where K_E is the dielectric constant or relative permitivity of the host medium here assumed to be a dielectric, not a conductor, eg. metal. The quantity

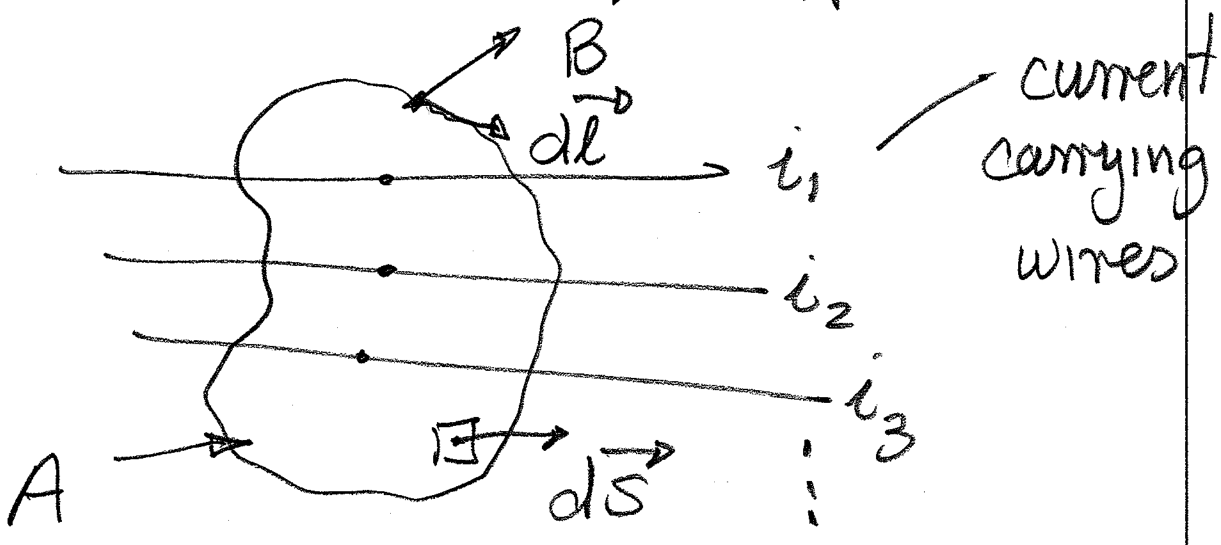
$$\epsilon = K_E \epsilon_0 \; ; \quad n^2 = K_E$$

is called the electric permittivity of the host medium.

Ampere's law ($\mu_0 = 4\pi \times 10^{-7}\ N \cdot s^2/c^2$)

We shall start from the circuit law for static magnetic fields (μ_0 is the permeability of free space)

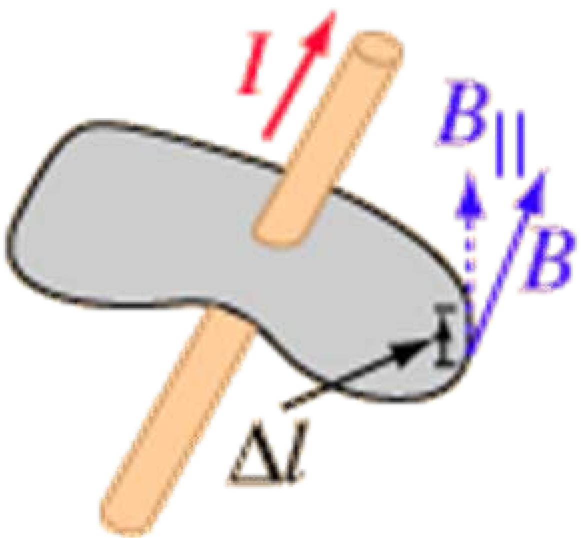
$$\oint_C \vec{dl} \cdot \vec{B} = \mu_0 \sum i = \mu_0 \iint_A \vec{J} \cdot \vec{ds}$$



Ampere's law

- For a single wire carrying a steady current I

$$\oint_C d\vec{\ell}\bullet\vec{B}(\vec{r}) = \mu_0 I = \mu_0 \iint_A d\vec{S}\bullet\vec{J}(\vec{r})$$



$$\sum B_{||}\Delta l = \mu_0 I$$

relates the line integral of $\vec{B}$ around a closed loop C to the sum of the currents i that pierce the area A bounded by C.

i - current in wire

$\vec{J}$ - current density (current/area) $\times \hat{n}$

It was realized that even in the absence of current carrying wires, $\sum i = 0$, a non-zero value of

$\Rightarrow \vec{J} = 0$,

$$\oint_C \vec{dl} \cdot \vec{B}$$

could arise if C was pierced by a time-varying electric field!

"time-varying electric field leads to a magnetic field"

James Clerk Maxwell (see paper on web) accounted for this by postulating that a time-dependent electric field produces a "displacement current density"

$$\vec{J}_D = \epsilon \frac{\partial \vec{E}}{\partial t}, \quad \epsilon = K_E \epsilon_0$$

to be added to $\vec{J}$ from the currents i. This leads to Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \iint_A \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

- Reduces to circuit law for statics $(\partial \vec{E}/\partial t) = 0$.
- Even with $\vec{J} = 0$, a time varying $\vec{E}$ generates a $\vec{B}$-field.

Maxwell's equations

We now write Maxwell's equation in integral form as you will have encountered them in Phys. 241. Here we assume a dielectric medium and set $\vec{J}=0$, $\sum i=0$, $\sum q=0$, $\epsilon=\epsilon_0 K_E$, no free charges or currents:

$$\oint\!\!\oint_A \vec{E}\cdot d\vec{S} = \oint\!\!\oint_A \vec{B}\cdot d\vec{S} = 0 \quad \text{(Gauss)}$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\iint_A \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \epsilon \iint_A \left(\frac{\partial \vec{E}}{\partial t}\right)\cdot d\vec{S}$$

Next we see how this yield wave eqns.