

Two beams (same source/wavelength/frequency):

Two-wave interference of 3D plane waves (similar to that of 1D)

$$\psi = A \cos[\vec{k}_1 \cdot \vec{r} - \omega t + p_1] + B \cos[\vec{k}_2 \cdot \vec{r} - \omega t + p_2]$$

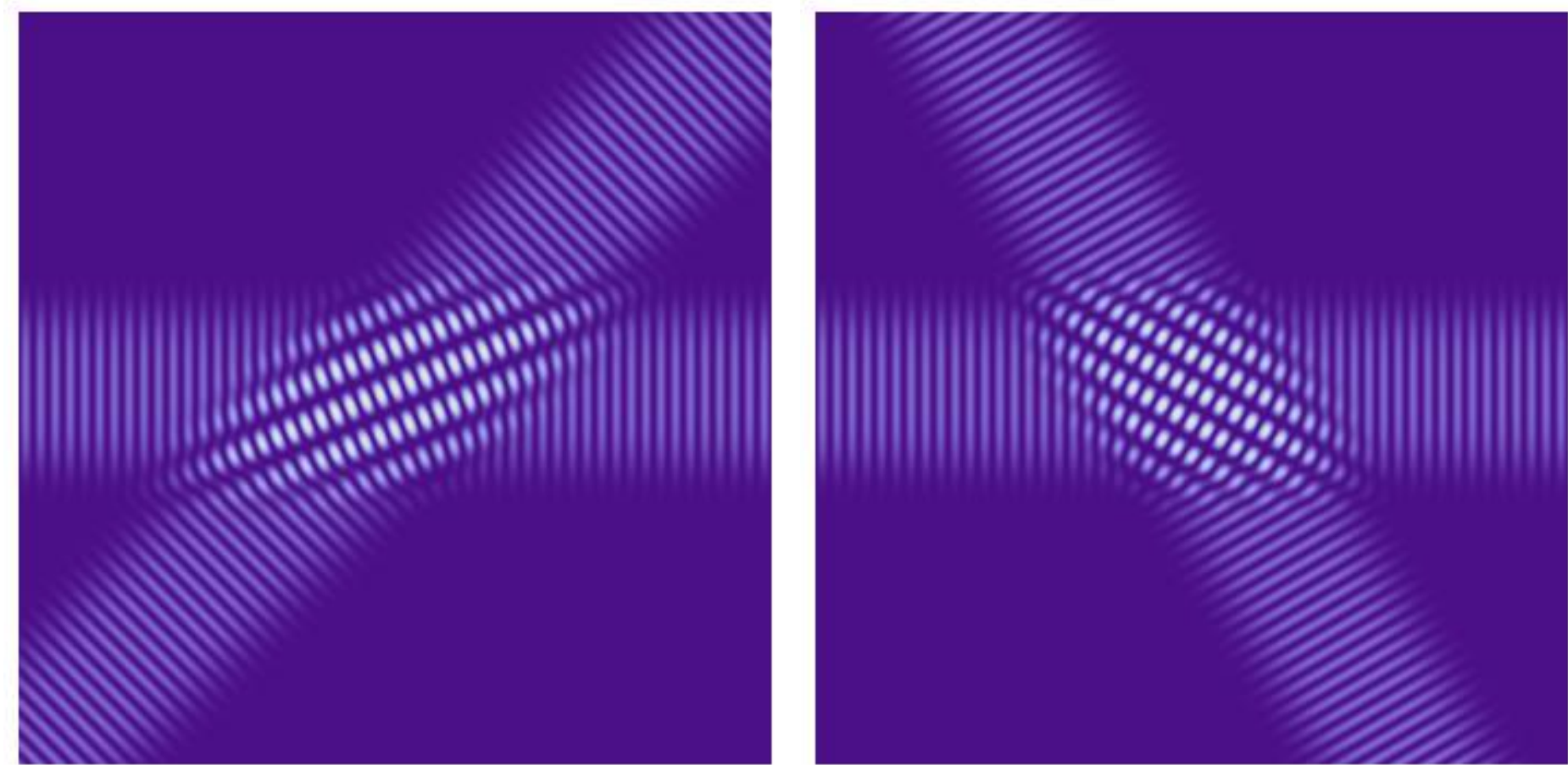
Resulting pattern = Envelope (slow change) \times Carrier (fast change)

$$\psi = \text{Re}\{\psi_c\} \quad , \quad \psi_c = [(A + B) \cos D + i(A - B) \sin D] \exp[iS] \quad ,$$

Properties of the interference pattern controlled by:

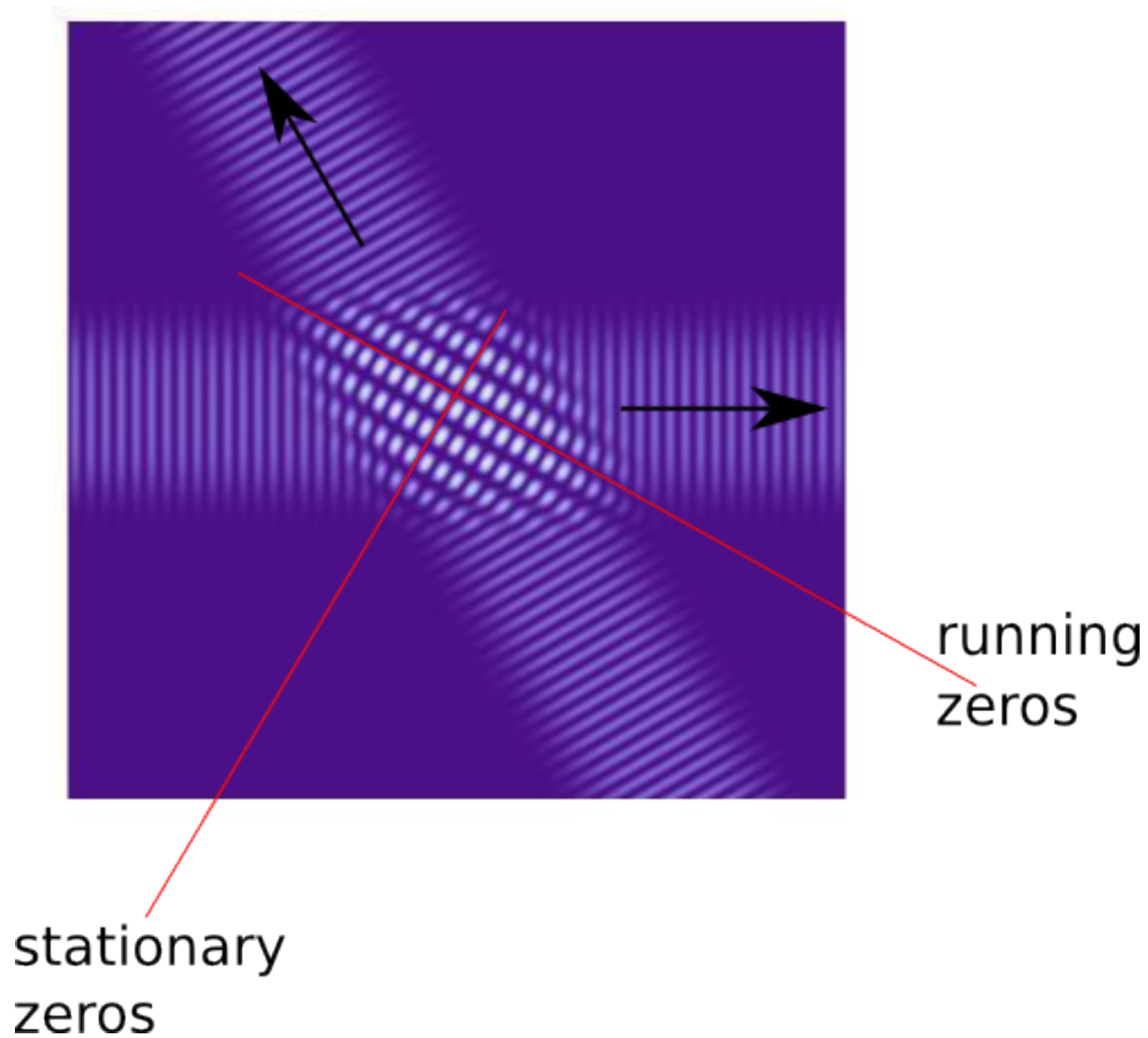
$$\text{Spread of parameters:} \quad D = (P_1 - P_2)/2 = \frac{1}{2}(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} + (p_1 - p_2)/2$$

$$\text{Average of parameters:} \quad S = (P_1 + P_2)/2 = \frac{1}{2}(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} + (p_1 + p_2)/2 - \omega t$$

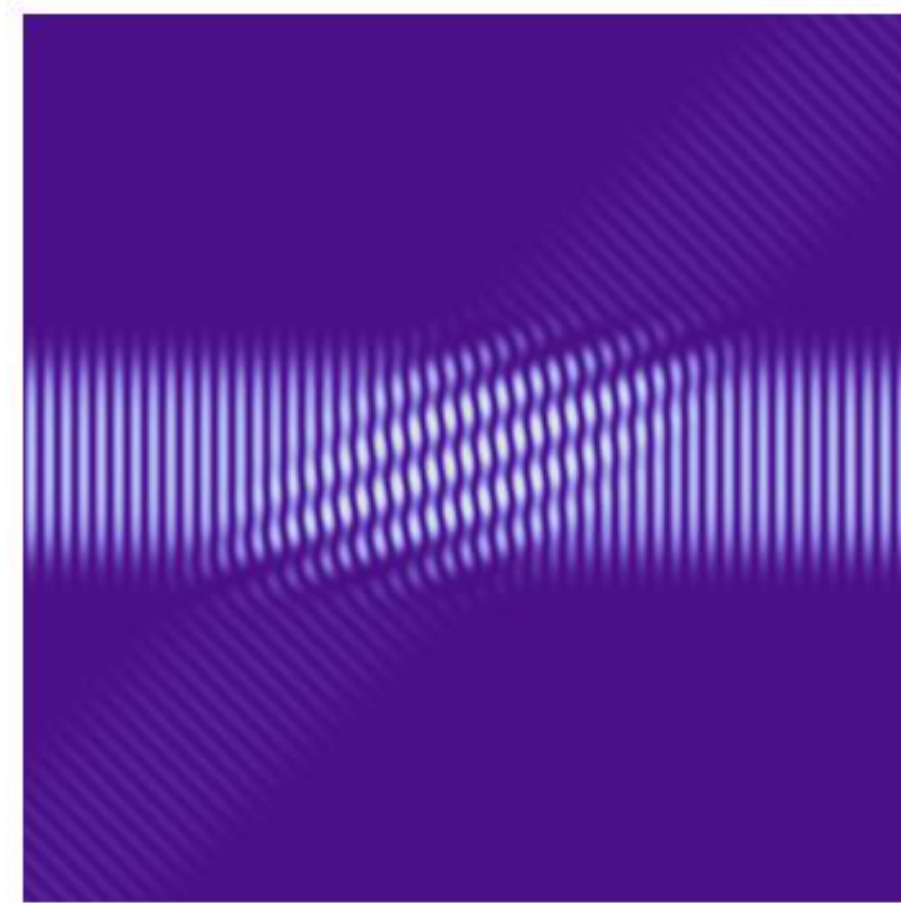


Two-wave interference of waves with equal wavelength (and thus equal frequencies), and amplitudes. Waves intersect at 45 degrees on the left, and at 120 degrees in the right panel. Note two kinds of zeros in the interference pattern.

This is a **snapshot**. A real detector would “see” cycle-averaged intensity. How would these pictures look?



Q: This is a case of **equal frequencies**. What happens to the stationary zeros if the frequencies are just slightly different?



Here we have a superposition of two waves with **unequal intensities**. Note that running zeros remain sharp. On the other hand, “envelope” zeros disappeared — this is because waves can not cancel each other completely.

Take-away:

In a two-wave interference pattern:

- two kinds of zeros (dark lines) in a snapshot (frozen time)
- one family of zeros moves with the carrier (i.e. very fast)
- one family of zeros is stationary
- running zeros disappear when observed with a real detector
- stationary zeros disappear if the waves have unequal amplitudes
- what is needed for contrast in interference patterns:
 - similar amplitudes
 - equal frequencies/wavelength