Name:	

OPTI 310, Fall 2017

Final Exam

Prof. M. Kolesik

Dec. 13, 2017.
10:30 am - 12:30 pm

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer ALL questions. Show supporting arguments — unjustified answers receive reduced credit!

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3 \times 10^8$ m/s $\qquad \hbar = 1.05 \times 10^{-34}$ Js

$\frac{\pi w_0^2}{\lambda} \qquad \frac{\pi w_0^2}{2}$

$\frac{1}{2}\epsilon_0 c n E_0^2$

$\sqrt{1\lambda L} \qquad \sqrt{2\lambda L} \qquad \sqrt{3\lambda L}\ldots \qquad L = \left(\frac{1}{h} + \frac{1}{h'}\right)^{-1}$

$\nu = \frac{c}{\lambda} = N\nu_{FSR} \qquad \mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} \qquad F = \frac{4R}{(1-R)^2} \qquad \frac{c}{2d} \qquad RP = N\mathcal{F}$

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 \quad , \quad \beta = \frac{\pi w \sin\Theta}{\lambda} \quad , \quad \gamma = \frac{\pi d \sin\Theta}{\lambda}$$

$$U_p \sim |U_1|/2 + (|U_1|/2 - |U_2| + |U_3|/2) + (|U_3|/2 - |U_4| + |U_5|/2) + \ldots \sim |U_1|/2$$

Score:	/out of 60 possible points

1. (20pts) This problem covers a variety of topics from the class. No long calculations are required.

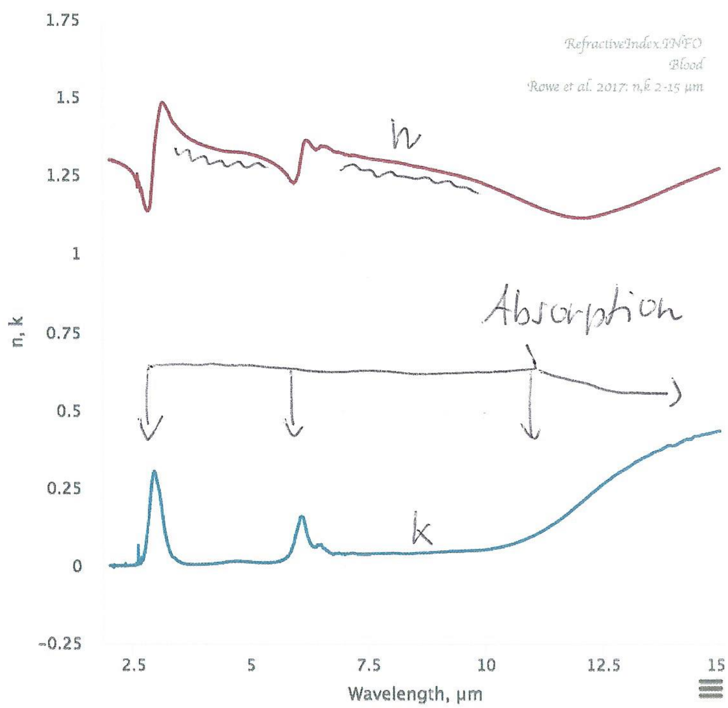
(a - 3pts) Identify one physical example of a transverse wave and one physical example of a longitudinal wave. Also indicate if ocean waves can be classified either as transverse or longitudinal, justifying your answer.

T: light, sound in solids

L: sound in gases

Ocean waves: neither

(b - 4pts) This figure shows real and imaginary parts of index of refraction of human blood



1) Which curve shows the real and which the imaginary part of the refractive index?
2) Identify two regions in which this liquid exhibits *significant* absorption. see ↓
3) Identify two regions in which it exhibits normal dispersion. see ~
4) Which part of this picture shows the visible region? none

(c - 2pts) This problem deals with the one-dimensional wave equation. Demonstrate that function $f(x,t) = \exp(\cos(x+2t))$ satisfies a wave equation, and determine the velocity and direction of this wave propagation.

i) $f(x,t) = F(x+vt)$; $v = 2$ & $F(\cdot) = \exp[\cos[\cdot]]$

ii) $\underbrace{\text{direct}}_{\text{long}}$ calculation : $\underbrace{\partial_{xx} f(x,t) \; ; \; \partial_{tt} f(x,t)}_{\text{into:}}$

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \phi$$

(d - 2pts) Describe the design of an anti-reflection coating. What are the guiding principles to choose the thickens and the material of the coating layer?

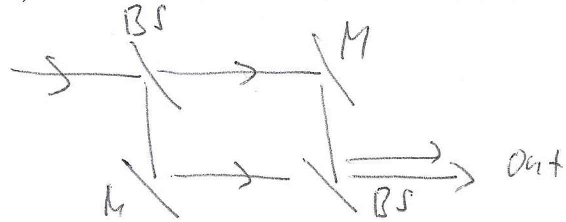
i) thickness = $\frac{\lambda}{n_c 4}$ ii) : $n_c = \sqrt{n_{substrate} \cdot n_{outside}}$

(e - 1pts) Describe the design of a highly reflective mirror based on the multilayer structure. How should you choose the thickness of the layers?

thickness = (always) = $\frac{\lambda}{4n}$

design = high / low / high / low / ...

(e - 2pts) Sketch the scheme of a Mach-Zehnder interferometer. Give an example for its application.

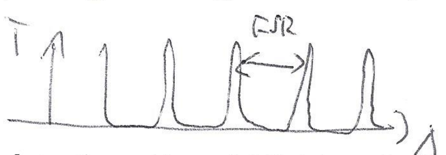


Application: switch, sensor

(e - 1pts) Explain the notion of the free spectral range of the Fabry-Perot interferometer.

i) "distance" between resonances

ii)



(e - 2pts) Explain the notion of "division of amplitude" and "division of wavefront." Give one example for each.

Ampl: Mach Zehnder

Wavefront: Two-Slit

(h - 3pts) Describe what a quarter-wave plate does to the polarization state of a linearly polarized beam in two cases:
A) the oscillation direction of the incident light is aligned with the fast or slow axis of the plate
B) the oscillation direction makes 45 degree angle with the axes of the plate

A) nothing

B) makes circularly polarized light

2. (10pts) The transmittance for a Fabry-Perot illuminated at normal incidence is given by the Airy function

$$\mathcal{T} = \left(\frac{I_T}{I_0}\right) = \frac{T^2}{1 + R^2 - 2R\cos(\Delta)} = \frac{T^2}{(1-R)^2}\frac{1}{1 + F\sin^2(\Delta/2)}, \tag{1}$$

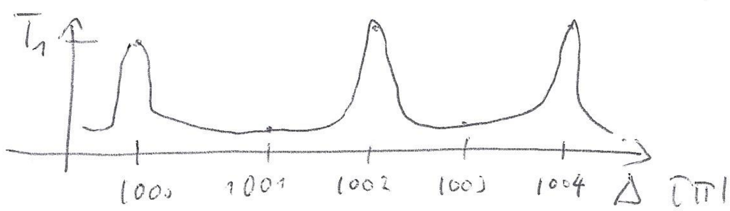
where $R = (1 - T)$ is the mirror reflectance (neglecting losses), and $\Delta = 2kd$ is the phase difference, d being the spacing between the mirrors. For this question set the mirror reflectance $R = 0.9$ and incident wavelength $\lambda = 0.5$ microns.

(a - 1pt) Starting from the Airy function prove that under conditions of resonance the internal intensity I_i inside a Fabry-Perot is a factor $(1/T)$ times larger than the incident intensity I_0.

$$E_T = \frac{t^2 E_0}{1 - r^2} = t\, E_{INT} \Rightarrow E_{INT} = \frac{t}{1-r^2} E_0 = \frac{\sqrt{T}}{T} E_0$$

$$I_T \propto |E_T|^2 \quad I_T \sim \frac{1}{T}|E_i|^2 \sim \frac{1}{T} I_0 \checkmark$$

(b - 3pts) Sketch the Fabry-Perot transmittance as a function of the phase difference for $\Delta = 1000\pi - 1004\pi$ indicating key features such as the spacing Δ_F between adjacent transmittance maxima, and the values of the phase difference at the minima and maxima of the transmittance (i.e. properly label the horizontal axis).



(c - 3pts) The width of the Fabry-Perot resonances $\delta\Delta$ is given by Δ_F divided by the Fabry-Perot finesse. By utilizing this information obtain an expression for the width of the Fabry-Perot resonances $\delta\nu$ when the transmittance is plotted as a function of the temporal frequency ν. Your answer should involve the free spectral range and finesse of the Fabry-Perot.

$$\mathcal{F} = \left(\frac{\partial\nu}{\Delta\nu}\right)^{-1} = \left(\frac{\partial\nu}{\frac{c}{2d}}\right)^{-1} \Rightarrow \partial\nu = \underbrace{\frac{c}{2d}}_{FSR}\frac{1}{\mathcal{F}}$$

(d - 3pts) Calculate the finesse, the free-spectral range, and the order of interference N for the Fabry-Perot assuming $d = 0.25$ mm.

$$\mathcal{F} = \frac{\pi}{2}\sqrt{F} = \pi\frac{\sqrt{R}}{1-R} \qquad R = 0.9 \; \ldots$$

$$FSR = \frac{c}{2d}$$

$$N\lambda = 2d \Rightarrow N = \ldots$$

3. (10 pts) In this problem, consider electromagnetic field as described by Maxwell equations in free space.
(a - 2pts) Finish the equation:

$$\partial_y E_z - \partial_z E_y = \ldots \qquad -\partial_t B_x$$

(b - 2pts) Finish the equation:

$$+\partial_t D_z = \ldots \quad = (\nabla \times \vec{H})_z = \partial_x H_y - \partial_y H_x$$

(c - 2pts) The fundamental invariants of the electromagnetic field are scalar quantities given as

$$P = \vec{B}.\vec{B} - \frac{1}{c^2}\vec{E}.\vec{E} \quad \text{and} \quad Q = \vec{B}.\vec{E}$$

The special significance of these quantities is that they have the same value in all coordinate systems (e.g. they do not change when the observer changes her velocity). Based on your knowledge of the properties of electromagnetic plane waves, decide what values P and Q attain for an arbitrary plane wave.

$P = 0 \Leftarrow$ equal E and M energy density

$Q = 0 \Leftarrow$ EM waves are transverse & $\vec{E} \perp \vec{B}$

In the following, consider the following plane-wave solution for the magnetic field

$$\vec{B}(\vec{r},t) = \hat{j}B_0 \cos\left(\frac{2\pi}{\lambda}\frac{x+z}{\sqrt{2}} - \omega t + \pi/4\right),$$

where ω is the field angular frequency and lambda is the wavelength

(d - 1pt) Specify the complex representation of this field

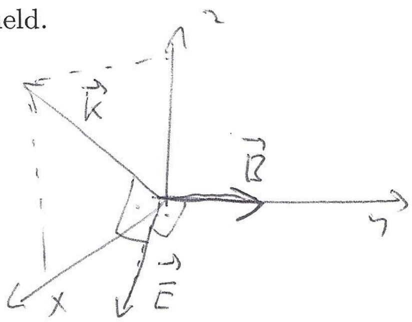
$$\vec{B} = \hat{j} B_0 \exp\left[i\left(\frac{2\pi}{\lambda}\frac{x+z}{\sqrt{2}} - \omega t + \pi/4\right)\right]$$

(e - 1pt) Determine λ in terms of the angular frequency.

$$\lambda = \frac{2\pi c}{\omega}$$

(f -1pts) Determine the electric field corresponding to the given magnetic field.

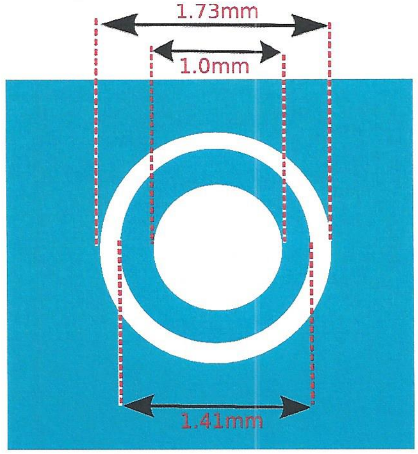
$$\vec{E} = \hat{e} B_0 c \exp[\ldots] \qquad \hat{e} = \frac{\hat{i} - \hat{k}}{\sqrt{2}}$$



(g - 1pt) Calculate the time-dependent Poynting vector from the real fields obtained above

$$\vec{S} = \underbrace{\frac{\hat{i} + \hat{k}}{\sqrt{2}}}_{\text{direction}} \underbrace{|S|}_{\text{magnitude}} \qquad |S| = c\varepsilon_0 E_0^2 \cos^2(\ldots)$$

4. (10pts) This problem deals with Fresnel diffraction. An infinite screen shown in the figure has two concentric openings as indicated in white. The screen is illuminated by a point source located on axis in the distance of 1/2 meter from the screen. The observation point is located on axis 1/2 meter from the screen. The light source wavelength is $\lambda = 1$micron.



(a - 2pts) Explain what is Fresnel zone.

FZ = area with approximately same phase

(b - 2pts) Calculate the radia of the first five Fresnel zones

$R_n = \sqrt{n\lambda L}$ $\quad L = \left(\frac{1}{h} + \frac{1}{h'}\right)^{-1} = \left(\frac{2}{m} + \frac{2}{m}\right)^{-1} = \frac{1}{4}m \leftarrow h = \frac{1}{2}m \quad h' = \frac{1}{2}m$

$R_n = \sqrt{n}\sqrt{10^{-6}\, 25\, 10^{-2} m^2} = \sqrt{n}\, 5.10^{-4} m$

$R_1 = 0.5mm$, $\quad R_2 = 0.5\sqrt{2}\, mm$, $\quad R_3 = \sqrt{3}\, 0.5mm$, $\quad R_4 = 1mm$, $\quad R_5 = \sqrt{5}\, 0.5mm$

(c - 2pts) Identify which Fresnel zones are illuminated, and say if the light from these zones interferes constructively or destructively.

$\varnothing\, 1mm = R_1$ $\quad \varnothing\, 1.41mm = \sqrt{2}R_1 = R_2$ $\quad \varnothing\, 1.73mm = \sqrt{3}R = R_3$

illuminated zones ≈ 1 + 3

(d - 2pts) If the illumination by the source without the screen would be I_0, what is the intensity at the observation point?

1+3 are in phase . $U = U_1 - \varnothing + U_3 - \varnothing \ldots \doteq 2U_1$

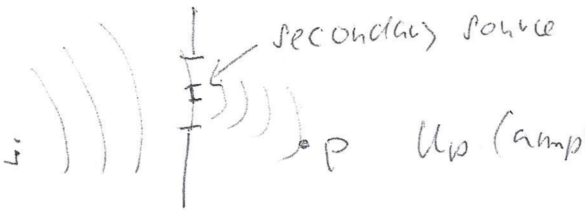
$I \sim U^2 = 4U_1^2$ $\quad U_1^2 \sim 4I_0 \Rightarrow I \sim 16 I_0$

(e - 2pts) If you want to further increase the intensity at the observation point by cutting one more concentric opening from the screen — how should you choose its dimensions? What will be the resulting intensity?

next open starts at R_4 ends at R_5

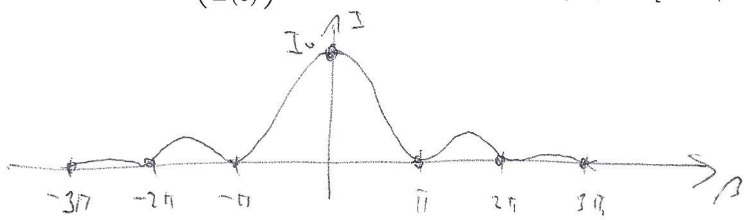
5. (10 pts) This question deals with Fraunhofer diffraction of a plane-wave electromagnetic field by an aperture.

(a - 3pts) Using a sketch and accompanying words describe the ideas behind the Huygens-Fresnel principle that underlies diffraction of an incident plane-wave by an aperture.



U_P (amplitude at observation point) $= \sum_i$ secondary sources

(b - 3pts) The one-dimensional Fraunhofer diffraction pattern produced by a plane-wave incident on a slit of width b is $E(\beta) = E(0) sinc\,(\beta)$, with $\beta = (\frac{kb}{2})\sin\theta$, and $\sin\theta = Y/R$ the angular deviation from the propagation axis, R being the distance from the aperture to the observation screen, and Y the transverse position on the screen. Sketch $\left(\frac{E(\beta)}{E(0)}\right)$ versus β over the range $\beta = [-3\pi, 3\pi]$ indicating key features.



(c - 2pts) Obtain an expression for the diameter D of the central peak of the intensity profile from part (b) as the distance between the closest two zeros surrounding the central peak. Your expression should involve R, λ and b.

$$\pi = \beta_1 \Rightarrow \pi = \frac{kb}{2}\sin\theta = \frac{2\pi}{2\lambda} b \frac{Y}{R} = \frac{\pi b}{\lambda}\frac{D}{2R} \Rightarrow D = \frac{2R\lambda}{b}$$

(d - 1pt) Based on your answers from parts (b) and (c) explain the observation that the diffracted intensity profile expands with increasing distance from the aperture to the observation screen whilst the intensity of the central peak decreases.

D growt mit distance R

Larger illuminated area means Lower intensity

(e - 1pt) According to wave theory the far field diffraction pattern due to the slit has an oscillatory intensity pattern that expands spatially with increasing propagation distance. Discuss whether or not these observations are in conflict with the predictions of geometrical optics.