Name:	
Score:	(see next page) /out of 40 possible points

OPTI 310, Fall 2016 **Mid-Term Exam 1** Prof. M. Kolesik

In-class exam, Friday Sept. 30, 2016.
11:00-11:50 am

Notes for the exam:
1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.
2. Answer all questions.
3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.
4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.
5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$c = 3 \times 10^8$ m/s. (OK to use this value)

$\epsilon_0 = 8.854187817... \times 10^{-12}$ F/m or A s/(V m) (Ok to approximate)

$\mu_0 = 4\pi 10^{-7}$ H/m or V s/(A m)

$\hbar = 1.054571800(13) \times 10^{-34}$ Js (OK to approximate with 1×10^{-34}...)

$$\frac{\pi w_0^2}{2} \quad , \quad \frac{\pi w_0^2}{\lambda} \quad , \quad w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

$$\nabla \times \nabla \times \vec{A} = \nabla\nabla.\vec{A} - \Delta\vec{A}$$

$$\vec{E} \times \vec{H} \quad , \quad \frac{1}{\mu_0}\vec{E} \times \vec{B} \quad , \quad \frac{1}{2}cn\epsilon_0 E_0^2$$

$$U(x - vt) + V(x + vt)$$

Score:	/out of 40 possible points

P1: (10pts) This problem deals with solutions to three dimensional wave equation. We will assume that waves propagate with the speed of light, but they are scalar (i.e. not vector) waves.

A) (2pts) Give the three-dimensional wave equation obeyed by the waves just described.

B) (4pts) We will consider the following function for $0 \leq x \leq a$ and $0 \leq y \leq a$:

$$\psi = \sin(\pi x/a)\sin(\pi y/a)\cos(k_z z - \omega t)\ .$$

Such a function can approximate the electric field of a wave propagating in a rectangular waveguide with square cross-section of size $a \times a$. For this to be a solution to the wave equation, a dispersion relation must by satisfied between k_z and ω. By direct calculation, inserting the solution into the wave equation, determine this dispersion relation.

C) (2pts) We could associate with this wave wavenumbers $k_x = \pi/a$ and $k_y = \pi/a$. Using the result from the previous part, show that the triple k_x, k_y, k_z then satisfies the same dispersion relation as waves propagating in vacuum. This provides a sanity check for your solution(s).

D) (1pt) The given function is real. Describe in words our convention for the complex representation, and write down the complex representation of ψ. (This can be done in different ways. Here, I do not want a decomposition into plane waves, so you can keep the part dependent on x and y as a real "amplitude" of the wave that propagates along z).

E) (1pt) Working from the last term in ψ, give an expression for the phase velocity along axis z (denote it v_p). Write v_p first as a function of ω and k_z. Then insert k_z obtained in part B) and decide if v_p is greater or smaller than speed of light.

P2: (10pts) This problem deals with Maxwell and wave equations in free-space.

A) (2pts) Write down Maxwell equations for free space, expressed in terms of $\vec{E}$ and $\vec{B}$. You may use the form employing operator ∇ in this part. Identify Ampere, Faraday, and Gauss laws.

B) (3pts) Starting from the Faraday law, derive the wave equation for the electric field.

C) (3pts) Write down all four Maxwell equations, this time in a component form, i.e. expressed in terms of E_x, E_y, E_z and B_x, B_y, B_z.

D) (1pt) Describe the meaning of the Poynting vector.

E) (1pt) Describe what is the time-averaged Poynting vector. Identify a formula for its value in an electromagnetic plane wave with the electric field amplitude E_0.

P3: (10pts) This problem concerns properties of electromagnetic plane-waves. Assume propagation in free space.

Which of these pairs **can** represent an electromagnetic plane wave? If it can, specify all sufficient conditions that must be satisfied. If a pair is not compatible with Maxwell equations, give at least one reason why. (2pts each)

A)

$$\vec{E} = \hat{\imath} E_0 \exp[i(kz - \omega t)] \qquad \vec{B} = \hat{\jmath} B_0 \exp[i(kz - \omega t)]$$

B)

$$\vec{E} = \hat{k} E_0 \cos[kx - \omega t] \qquad \vec{B} = \hat{k} E_0/c \cos[kx - \omega t]$$

C)

$$\vec{E} = \hat{k} E_0 \cos[2\pi/\lambda((x+y)/2 - ct)] \qquad \vec{B} = \hat{\jmath} E_0/c \cos[2\pi/\lambda((x+y)/2 - ct)]$$

D)

$$\vec{E} = \hat{\jmath} E_0 \sin[k(x - ct)] \qquad \vec{B} = +\hat{k} E_0/c \sin[k(x - ct)]$$

E) If the following is the electric field in a plane wave

$$\vec{E} = \hat{k} c B_0 \cos[2\pi/\lambda(x+y)/\sqrt{2} - ct)] \ ,$$

what is the corresponding magnetic field?

P4: (10pts) This problem concerns Gaussian beams and notions of energy and momentum transfer by photons. A laser beam is transmitted by a telescope that produces a Gaussian beam with the beam waist of $w_0 = 1$m. The beam is directed toward a GPS satellite at a distance of $D = 20000$km. The laser wavelength is $\lambda = 694.3$nm. (2pts each)

A) Explain the meaning of the parameter w_0 and its relation to the effective area A of the beam.

B) Explain what is Rayleigh range. Calculate its value for the given parameters.

C) Calculate the laser beam spot size $w(z = D)$ at the distance of the satellite. Compare D to the Rayleigh range obtained above, and based on this decide if one can simplify the formula by neglecting unity under square root?

D) Assume that the laser emits a pulse with energy $E = 3$J. How many photons are in a single pulse?

D) What is the total momentum of one laser pulse?