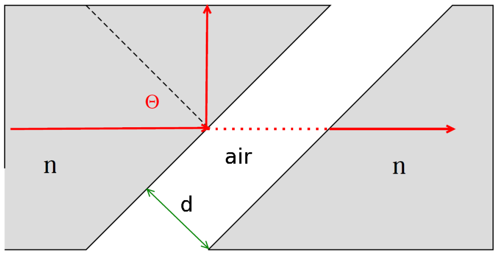
This problem illustrates that results derived for Fabry-Perot can be applied in various situations...

P0: Revisit "optical tunneling effect." The picture shows light "tunneling" through a narrow slit between two blocks of glass. For simplicity assume that the angle of incidence is 45 degrees, polarization is TE, and $n > \sqrt{2}$. Calculate the transmittance.



We know from our treatment of TIR, that the wave in the "forbidden territory" just beyond the glass-air interface decays exponentially. One therefore expects that the transmittance will be also exponentially small. Use this to simplify your calculation and derive only the leading (i.e. most important) term to describe the transmittance.

Hint 1:
The easiest way to solve this problem is to apply one of the Fabry-Perot related formulas for the transmission coefficient. However, care must be taken to use the right one!

Hint 2:
Once you identify the formula to start from, you need to figure out how to handle propagation in the air gap. Recall that our transfer matrix calculations suggested that what appears in the FP formulas and also in Fresnel formulas are just z components of the wave vector(s).

P0 Solution

The right formula to use is

$$t = \frac{E_T}{E_0} = \frac{1 - r^2}{e^{-i\phi} - e^{+i\phi} r^2} \qquad \phi = k_z d \equiv \delta/2$$

The reason that the other does not lead to the correct result is that the partial wave summation neglected the overall phase. However, in this case the wave traversing the gap suffers not only a phase change but also attenuation — which must not be neglected. Thus, *all* factors $e^{i\delta}$ must be accounted for, and the above formula does.

The next step is to realize that as in the TIR, k_z turn out to be purely imaginary. We calculate it the usual way:

In glass:

$$k_\perp^2 + k_\parallel^2 = \frac{\omega^2 n^2}{c^2} \qquad k_\parallel = \frac{\omega n}{c} \frac{1}{\sqrt{2}}$$

In the air gap:

$$k_\parallel^2 + k_z^2 = \frac{\omega^2}{c^2}$$

Eliminate $k_\perp$ and calculate k_z:

$$k_z = i \frac{\omega}{c} \sqrt{\frac{n^2}{2} - 1}$$

We also need r. For that we simply apply the Fresnel formula for r_s, while keeping in mind that $\cos\theta_t$ is imaginary because the incidence angle is larger than critical.

$$r_s \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{\frac{n}{\sqrt{2}} - i\sqrt{\frac{n^2}{2} - 1}}{\frac{n}{\sqrt{2}} + i\sqrt{\frac{n^2}{2} - 1}}$$

Before inserting all "ingredients" into the transmission coefficient formula, we should figure out which term in it is exponentially small and which is exponentially large. It follows from:

$$e^{+i\phi} = \exp[-\frac{\omega}{c} d\sqrt{\frac{n^2}{2} - 1}] \qquad e^{-i\phi} = \exp[+\frac{\omega}{c} d\sqrt{\frac{n^2}{2} - 1}]$$

We can therefore neglect the second term in the denominator, and get

$$t = (1 - r^2)e^{+i\phi}$$

and the transmittance evaluates to:

$$T = |t|^2 = 4n^2 \frac{n^2 - 2}{(n^2 - 1)^2} \exp[-\frac{2\omega}{c} d\sqrt{\frac{n^2}{2} - 1}]$$

Sanity check (?)

The above result does not pass the following sanity check: When we take n closer and closer to $\sqrt{2}$, the given incidence angle of 45 degrees tends closer to critical. At the same time, T goes to zero, which is obviously a wrong behavior.

A) Explain why we do not expect $T \to 0$

B) Explain why this "test" must not be applied to the result for T.