This problem deals with identification of the polarization state, given the vector field of electric intensity. The approach here is straightforward:
1. project the field onto observation plane
2. switch to local time in arguments of functions describing $\vec{E}(\vec{r}, t)$
3. identify several special points for the tip of the electric field vector in the observation plane. Having these points, and knowing that the answer is 'ellipse' in general, tells you the type of the polarization state.

P0:
Identify the polarization state of the following waves:

A)

$$\vec{E} = \hat{\imath} E_0 \sin[2\pi(z/\lambda - \nu t)] + \hat{\jmath} E_0 \cos[2\pi(z/\lambda - \nu t)]$$

B)

$$\vec{E} = \hat{\imath} E_0 \sin[2\pi(z/\lambda - \nu t)] + \hat{\jmath} E_0 \sin[2\pi(z/\lambda - \nu t - 1/8)]$$

C)

$$\vec{E} = \hat{\imath} E_0 \sin[2\pi(z/\lambda - \nu t)] - \hat{\jmath} E_0 \sin[2\pi(z/\lambda - \nu t)]$$

Solution

A

Given field:

$$\vec{E} = \hat{\imath} E_0 \sin[2\pi(z/\lambda - \nu t)] + \hat{\jmath} E_0 \cos[2\pi(z/\lambda - \nu t)]$$

Step 1. Choose your observation plane, e.g. $z = 0$:

$$\vec{E} = \hat{\imath} E_0 \sin[2\pi(-\nu t)] + \hat{\jmath} E_0 \cos[2\pi(-\nu t)]$$

$$\vec{E} = \hat{\imath}(-1) E_0 \sin[2\pi\nu t] + \hat{\jmath} E_0 \cos[2\pi\nu t]$$

Step 2. Switch to "local time" $\tau = 2\pi\nu t = \omega t$. This "time" is nothing but phase, with 2π representing time of a single optical cycle:

$$\vec{E} = \hat{\imath}(-1) E_0 \sin[\tau] + \hat{\jmath} E_0 \cos[\tau]$$

Note that unlike phase $\phi = 2\pi(z/\lambda - \nu t)$, τ increases with time, that is why we can call it "local time."

Step 3. Now we find several points on the polarization ellipse. First, determine the "bounding box" in which this ellipse lives:

$$-E_0 \leq E_x \leq +E_0 \qquad -E_0 \leq E_y \leq +E_0$$

The point where the ellipse touches the bounding box are easy to determine. For example, E_x attains the box when $\tau = \pm\pi/2$. Let us figure out the tip of $\vec{E}$ for these times:

$$\vec{E}(\pm\pi/2) = \hat{\imath}(-1) E_0 \sin[\pm\pi/2] + \hat{\jmath} E_0 \cos[\pm\pi/2]$$

$$\vec{E}(\pm\pi/2) = \hat{\imath}(-1)E_0(\pm 1) + \hat{\jmath}E_0 * 0$$
$$\vec{E}(-\pi/2) = (+1)\hat{\imath}E_0 \qquad \vec{E}(+\pi/2) = (-1)\hat{\imath}E_0$$

Now let us do the same for points at which E_y is extremal. I look at the argument in the second term to guess these:

$$\vec{E}(k\pi) = \hat{\imath}(-1)E_0 \sin[k\pi] + \hat{\jmath}E_0 \cos[k\pi] \qquad k = 0, 1$$
$$\vec{E}(0) = (+1)\hat{\jmath}E_0 \qquad \vec{E}(\pi) = (-1)\hat{\jmath}E_0$$

So we have found the tip locations for times corresponding to

$$-\pi/2, 0, \pi/2, \pi$$

and they tell us (draw a sketch) that we deal with circular polarization (ellipse is a circle), with the tip rotating counter-clockwise, so this is Left-Hand Circular (LHC) polarization state.

B Given field:

$$\vec{E} = \hat{i}E_0 \sin[2\pi(z/\lambda - \nu t)] + \hat{j}E_0 \sin[2\pi(z/\lambda - \nu t - 1/8)]$$

Transform to τ-language for $z = 0$:

$$\vec{E} = \hat{i}E_0 \sin[2\pi(-\nu t)] + \hat{j}E_0 \sin[2\pi(-\nu t - 1/8)]$$

$$\vec{E} = (-1)\hat{i}E_0 \sin[\tau] + (-1)\hat{j}E_0 \sin[\tau + \pi/4]$$

Evaluate a few points:

$$\vec{E}(\tau = 0) = (-1)\hat{j}E_0 \sin(\pi/4) = (-1)\hat{j}E_0/\sqrt{2}$$

$$\vec{E}(\tau = \pi/4) = (-1)\hat{i}E_0 \sin[\pi/4] + (-1)\hat{j}E_0 \sin[\pi/2] = (-1)\hat{i}E_0/\sqrt{2} + (-1)\hat{j}$$

$$\vec{E}(\tau = \pi/2) = (-1)\hat{i}E_0 \sin[\pi/2] + (-1)\hat{j}E_0 \sin[\pi/2 + \pi/4] = (-1)\hat{i}E_0 + (-1)\hat{j}E_0/\sqrt{2}$$

From the three points, connected in the order of increasing time (draw sketch) , we see that the figure is an ellipse drawn in the clock-wise direction. So this polarization state is Right-Hand Elliptic.

C Given field:

$$\vec{E} = \hat{i} E_0 \sin[2\pi(z/\lambda - \nu t)] - \hat{j} E_0 \sin[2\pi(z/\lambda - \nu t)]$$

Projected to local time language at $z = 0$:

$$\vec{E} = (-1)\hat{i} E_0 \sin[\tau] - (-1)\hat{j} E_0 \sin[\tau]$$

Simplifies to:

$$\vec{E} = (\hat{j} - \hat{i}) E_0 \sin[\tau]$$

So this oscillates along a line direction given by $\hat{j} - \hat{i}$ which is at 45 degrees, in NW-SE direction. This is therefore a linearly polarized wave.