P1:

An electromagnetic plane wave propagates along the direction given by a vector $(-1,-1,-1)$. It is linearly polarized, with the direction of oscillation of the electric field given by a vector $(-2,-1,3)$. This wave is incident on a material interface from air. The medium-air boundary is given by the plane with the equation $y+z=0$. The refractive index of the material is $\sqrt{2}$.

- A Calculate the unit vector $\hat{k}$ in the direction of propagation
- B Calculate the unit normal $\hat{n}$ of the interface. Orient this vector such that it points into air.
- C Determine the plane of incidence. Recall that it is the plane given by the surface normal, and the incident wave propagation direction.
- D Calculate $\hat{e}_s$, the unit polarization vector for the s-polarized wave.
- E Calculate $\hat{e}_p$, the unit polarization vector for the p-polarized wave.
- F Using Fresnel formulas (e,g in this "symmetric form"),

$$r_\perp = \frac{n_i \cos\Theta_i - n_t \cos\Theta_t}{n_i \cos\Theta_i + n_t \cos\Theta_t} \qquad r_\| = -\frac{n_t \cos\Theta_i - n_i \cos\Theta_t}{n_t \cos\Theta_i + n_i \cos\Theta_t}$$

  determine the reflection coefficients for both polarizations.
- G Using these values, and relations for reflectance, together with the wave decomposition into s and p polarized components, determine the reflectance for the whole incident wave.

P2:

An electromagnetic plane wave propagates along the direction given by a vector $(0, -1, -1)$. It is linearly polarized, with the direction of oscillation of the electric field given by a vector $(1, -1, 1)$. This wave is incident on a material-air interface from a medium with a refractive index of $\sqrt{2}$. The medium-air boundary is given by the plane with the equation $x + y + z = 0$.

A Calculate the unit vector $\hat{k}$ in the direction of propagation

B Calculate the unit normal $\hat{n}$ of the interface. Orient this vector such that it points into air.

C Determine the plane of incidence.

D Calculate $\hat{e}_s$, the unit polarization vector for the s-polarized wave.

E Calculate $\hat{e}_p$, the unit polarization vector for the p-polarized wave.

F Calculate cosines of both the incident and the transmitted angle.

G Using Fresnel formulas (write the form you choose to use), determine the transmission coefficients for both polarizations.

H Determine the percentages of energy in the incident beam that propagates in the s and p polarized waves.

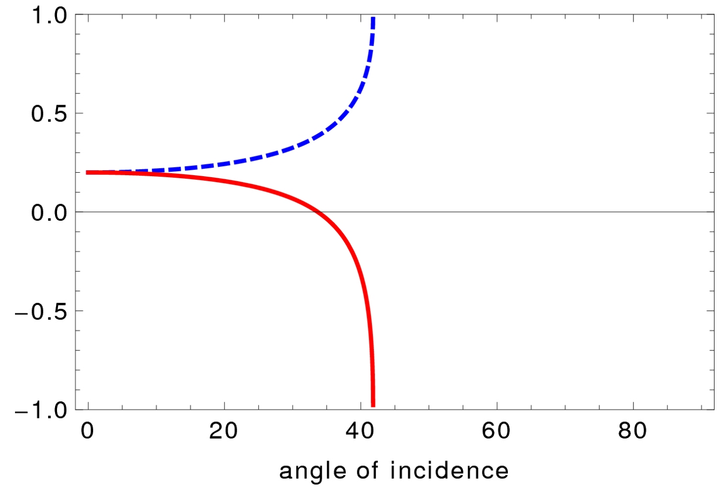
I Using these values, and relations for transmittance, determine what fraction of power of the whole incident wave is transmitted through the interface.

P3:

Write a Mathematica, Matlab or any other language program to solve the general problem of the same type as the previous two examples.

Assume that three (not necessarily normalized) vectors are given for the direction of propagation, direction of linear polarization (check that the two are orthogonal!), and the normal to the interface. Also, the "incident" and "transmitted" refractive index are known inputs.
Implement both the transmission and reflection quantities. Check energy conservation.

P4:



A Which (Fresnel formulae related) quantities are shown?

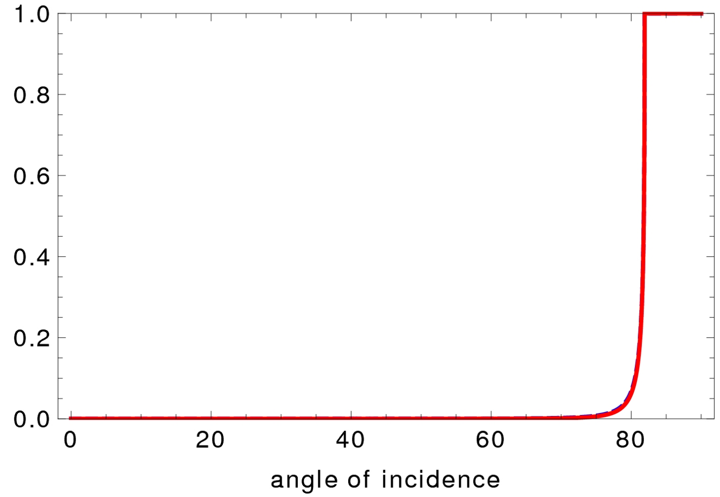
B Describe three different ways to determine the relative index of refraction for the interface in question.

C Which of the three seems most accurate in this case and why?

D Mark the critical and Brewster angles in the picture.

E Is this describing internal or external incidence?

P5:

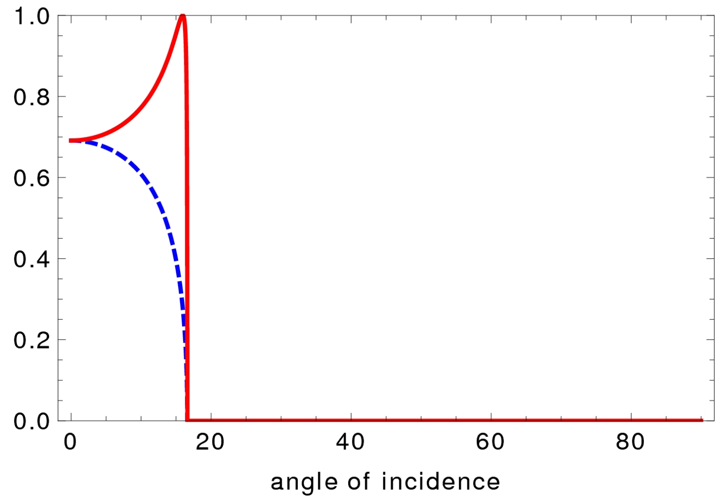


A There is two curves almost overlapping in this figure. Which quantities are shown?

B What can you say about the relative index of refraction? What physical situation it may represent?

C Where do you think is the Brewster angle in this picture? Give a qualitative argument supporting your answer.

P6:

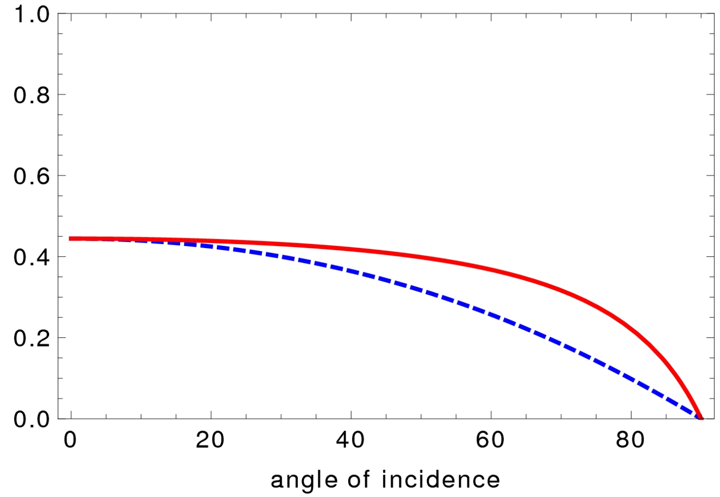


A Which quantities are shown?

B What can you say about the relative index of refraction? What physical situation it may represent?

C Which curve represents TE polarization?

P7:



A Is this internal or external incidence?

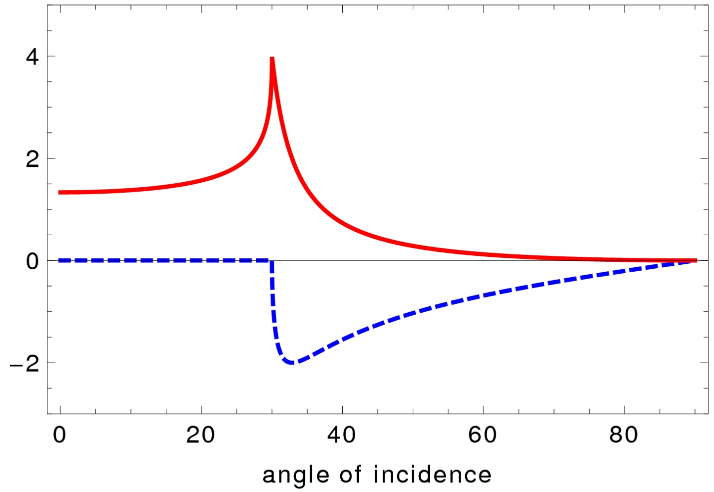
B Are there curves showing reflectivity or transmission coefficients?

C Estimate the relative refractive index.

D Evaluate r_s, r_p, t_s, t_p for the incident angle of 60 degrees (or so), and decide which curve belongs to which polarization.

E Extra: give an analytic argument that allows to decide the above question.

P8:



This picture shows real and imaginary part of a transmission coefficient.

A Is this internal or external incidence?

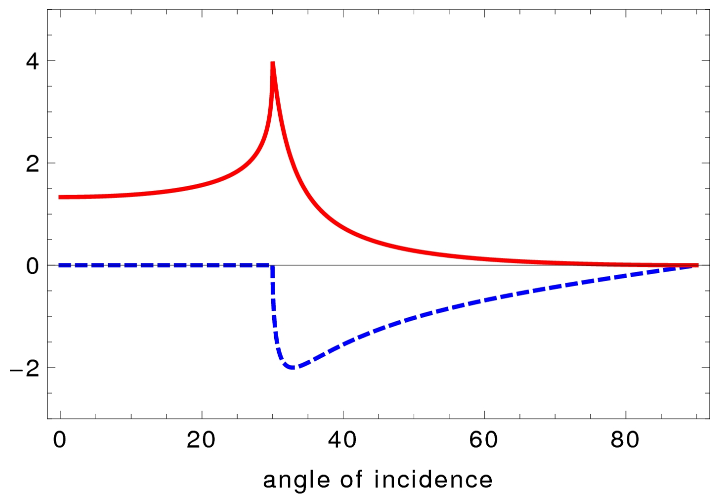
B Which curve represents the imaginary part?

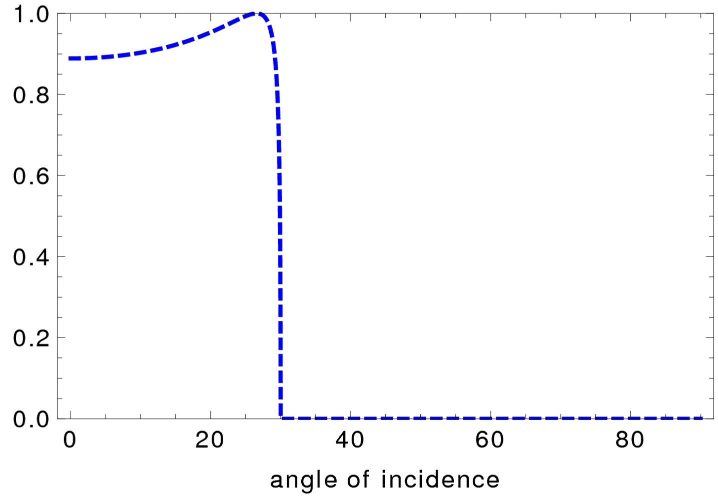
C Mark the critical angle.

D Estimate the relative refractive index.

E Show, approximately, where you expect the Brewster angle to be.

P9:





Explain the following "paradox":

The left picture is the same as in the previous example. It shows that a complex valued transmission coefficient exists for large angles beyond the critical. Picture on the right shows the corresponding transmittance, which is zero for $\theta_i > \theta_c$.
We have previously derived

$$T = |t|^2 \frac{n_t}{n_i} \frac{\cos\theta_t}{\cos\theta_i}$$

which seems to suggests that if t is a non-zero, complex-valued quantity for $\theta_i > \theta_c$, T should not be zero. So, strictly speaking there is something wrong with the above formula — what is it? How do you reconcile the two pictures (both correct)?

P10:

A)

The four numbers $\{4.84, 2.0, 1.0, -1.0\}$ represent values of reflection and transmission coefficients in some unknown order. Assign these values to r_s, r_p, t_p, t_s. Justify your answer.

B)

The same problem for $\{0.46, 0.36, -0.12, -0.64\}$