Practice problems: 1D and 3D waves, complex representation

1. Consider a one-dimensional scalar field $\psi(x,t)$ propagating along the x-axis with velocity v.

(a) Write down the one-dimensional wave equation obeyed by the scalar field $\psi(x,t)$. (2)

(b) Write down a solution for the harmonic wave $\psi(x,t)$ of (temporal) frequency ν, initial phase ε for $x = t = 0$, propagation number k, and amplitude A. (2)

(c) What is the wave velocity v in m/s for a harmonic wave of temporal frequency $\nu = 10^{15}$ s^{-1} and wavelength $\lambda = 0.3$ μm? (2)

(d) For your choice of harmonic wave solution in part (b), and assuming a transverse wave, sketch the spatial variation of $\psi(x,t)$ between $x = 0$ and $x = 2\lambda$ for $A = 1$ and $t = \varepsilon = 0$. (2)

2. This problem involves the complex plane-wave solution $\psi(\vec{r},t) = A\exp[i(\vec{k}\cdot\vec{r}-\omega t+\varepsilon)]$ of the three-dimensional wave equation. Assume that ε was chosen such that A is real-valued.

(a) Explain how one extracts the physical solution from the above complex plane-wave solution. (2)

(b) Based on your answer from part (a) write down the physical harmonic solution associated with the complex plane-wave solution. (2)

(c) What is the name and symbol for the quantity that determines the direction of propagation of the plane-wave solution? (2) Specify it for a case of a wave propagating at 45 deg with respect to the z-axis and at 90 deg w.r.t. the x-axis. Assume that the wave propagates in the vacuum and has the wavelength of 500nm.

(d) For $t = 0$ consider a surface which is transverse to the propagation vector $\vec{k}$ of the complex plane-wave solution. What can you say about the value of the complex plane-wave over such a surface? Specify one example of such a plane. (2)

(e) Consider two solutions ψ_1 and ψ_2 of the linear three-dimensional wave equation. Circle the combinations of the two solutions below that would be legitimate solutions ψ of the wave equation: (2)

$\psi = (\psi_1 + \psi_2)$, $\psi = \psi_1\psi_2$, $\psi = \frac{1}{2}(\psi_1 - \psi_2)$

$\psi = \psi_1/\psi_2$, $\psi = \frac{1}{100}(\psi_1 + i\psi_2)$, $\psi = \psi_1^2$