This is to illustrate in detail how to work with rotated matrices and transformation from one frame of reference to the other. Note that notation is subject to conventions (and therefore a "good opportunity" for errors). One can choose different orientation of the increasing rotation angle, and also different interpretation of the resulting rotation matrices (do we want rotate the coordinate system or the object?)...

Our choice here is that rotation angle is positive when anti-clockwise, and $R(\alpha)$ is the matrix that takes a vector from the original to the rotated coordinate system.

Problem: Consider a vertical polarizer followed by a quarter-wave plate rotated from the y-axis by 30 degrees. The latter is then followed by a horizontal polarizer. Describe in terms of Jones calculus.

Answer:

First let us fix the coordinate system. x, y without prime will refer to the LAB frame. Primed x' y' will denote the local (LOC) coordinate system connected with the QW plate. It is in LOC that we know the Jones matrix. This time it is

$$M' = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Solution 1: Let us assume we do remember the formula we have derived in the class (with c,s standing for cos and sin of the rotation angle):

$$M = R(-\alpha)M'R(\alpha) = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \begin{pmatrix} c^2 + is^2 & (1-i)cs \\ (1-i)cs & ic^2 + s^2 \end{pmatrix}$$

which gives the matrix M in the LAB given M' that we know in LOC. Since the polarizers are aligned with axes in LAB, they do not need transformation, and the composite Jones matrix will be:

$$\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} M \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \begin{pmatrix}c^2 + is^2 & (1-i)cs\\ (1-i)cs & ic^2 + s^2\end{pmatrix} \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}0 & (1-i)cs\\ 0 & 0\end{pmatrix}$$

Because of the first polarizer the Jones vector on the output will always be

$$\begin{pmatrix}(1-i)cs\\ 0\end{pmatrix}$$

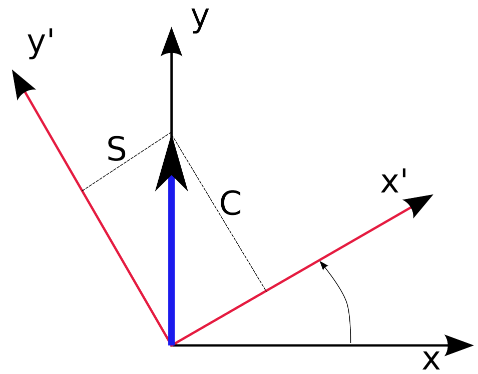
Solution 2: This time let us go step by step through the system. Start with the input Jones vector

$$V = \begin{pmatrix}0\\ 1\end{pmatrix}$$

This is in LAB, of course. Next, figure out how the same vector looks in LOC:

$$V' = R(\alpha)V = \begin{pmatrix}s\\ c\end{pmatrix}$$

If we did not remember if the matrix should carry $+\alpha$ or $-\alpha$, this is the point where one should sketch a picture and choose the right sign...



Next, we apply the matrix of the QW plate. This is easy since at this moment we are in LOC, so

$$V'_{afterHW} = M' \begin{pmatrix} s \\ c \end{pmatrix} = \begin{pmatrix} s \\ ic \end{pmatrix}$$

Now it is time to go back to LAB: Here we use the same matrix R, but with opposite sign of α because the rotation is to undo the first one:

$$V_{afterQW} = R(-\alpha) V'_{afterQW} = \begin{pmatrix} c & -s \\ +s & c \end{pmatrix} \begin{pmatrix} s \\ ic \end{pmatrix} = \begin{pmatrix} (1-i)cs \\ ic^2 + s^2 \end{pmatrix}$$

Finally we apply the second polarizer in LAB:

$$V_{out} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} V_{afterQW} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (1-i)cs \\ ic^2 + s^2 \end{pmatrix} = \begin{pmatrix} (1-i)cs \\ 0 \end{pmatrix}$$

Which solution is easier probably depends on the taste. The second is perhaps more resistant to errors related to wrong signs in α (which matrix I should use?), because one can check and choose "the right sign" in step one. Also note that checking this with a graphical approach is straightforward, and this is something one should always try to do.