This problem illustrates certain universal properties of far-field (i.e. Fraunhofer) diffraction patterns.

P0: Consider 1D Franuhofer diffraction through a slit. Following the derivation procedure used in the class,

A) Show what happens to the diffraction pattern when we use a plane wave that is incident not perpendicular but at angle γ on the screen.

B) What happens if the slit is moved up by a certain shift s? Is there a change visible in the intensity of the far field pattern?

C) Show that if the incident plane wave is normal to the screen, then the pattern has an inversion symmetry: If the angle in the far field is θ then the intensity is the same at $I(\theta)$ as for the opposite angle,

$$I(\theta) = I(-\theta)$$

D) This result may seem trivial for a single slit. Generalize it for an arbitrary collection of slits, for example for a pair with one wide and one narrow slit. In other words, show that even if the screen does not have up-down symmetry, the diffraction pattern does.

Note: We will later see that this symmetry property remain true also in 2D diffraction: Instead of up-down symmetry, we will speak about *inversion* symmetry.

P0 Solution:

A)

In this case the only modification is the wave amplitude in the integral:

$$E \sim \int_{-a/2}^{+a/2} dy A(y) e^{-ikyY/R}$$

(leaving out factors not important to answer this question)
The appropriate amplitude for the beam incident at angle γ is

$$A(y) = e^{ik_y y} \qquad k_y = k \sin\gamma = \frac{2\pi}{\lambda}$$

So the integral becomes:

$$E \sim \int_{-a/2}^{+a/2} dy e^{iky\sin\gamma} e^{-ikyY/R} = \int_{-a/2}^{+a/2} dy e^{-iky(Y/R - \sin\gamma)}$$

This is the same result we obtained for the normal incidence but with this replacement:

$$Y/R \to Y/R - \sin\gamma$$

for small angles (and none of this is indeed valid for not-so-small angles) we get:

$$\theta \to \theta - \gamma$$

This means that the whole diffraction pattern is angularly shifted by the same angle γ: The maximal intensity shifts to the angle given by the incident wave.

P0 Solution:

B)

This time the modification consists in moving the integration bounds in the Franuhofer diffraction formula:

$$E \sim \int_{-a/2}^{+a/2} dy \; 1 \; e^{-ikyY/R} \rightarrow \int_{-a/2+s}^{+a/2+s} dy 1 e^{-ikyY/R}$$

substitute: $y' = y + s$ to get

$$E \sim \int_{-a/2+s}^{+a/2+s} dy e^{-ikyY/R} = \int_{-a/2}^{+a/2} dy' e^{-ik(y'-s)Y/R} = e^{iksY/R} \int_{-a/2}^{+a/2} dy' e^{-iky'Y/R}$$

The last term is nothing but the original result multiplied by a phase factor with phase proportional to the spatial shift s.

$$e^{iksY/R}$$

This will of course not change $I = |E|^2$ in any way, so the far-field intensity pattern does not change.

P0 Solution:

C)
Write the intensity as a product of two diffraction integrals, complex conjugated to each other:

$$I(Y) \sim \int_S dy e^{-ikyY/R} \int_S dy' e^{+iky'Y/R}$$

then

$$I(-Y) \sim \int_S dy e^{+ikyY/R} \int_S dy' e^{-iky'Y/R}$$

which is the same as $I(Y)$ once we rename $y' \leftrightarrow y$.
So the diffracted intensity is the same at point Y and $-Y$ and the corresponding angles.

Note: You should see that this is straightforward to generalize to 2D diffraction, for which the symmetry is

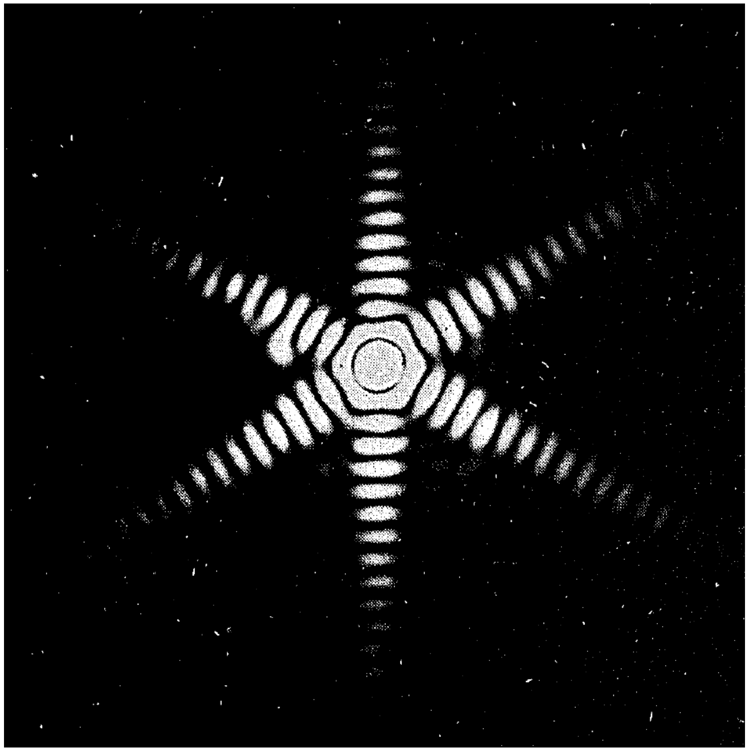
$$I(Y, Z) = I(-Y, -Z)$$

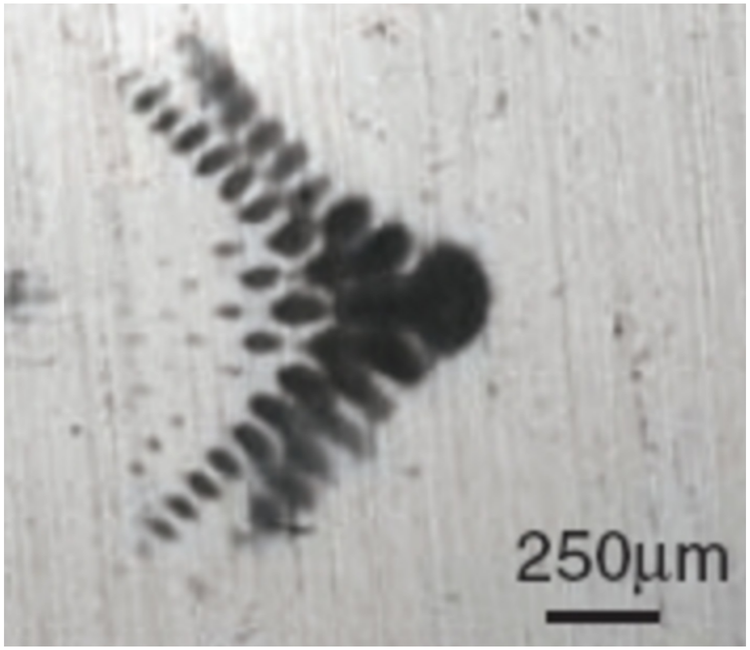
This is called inversion symmetry. It says that the intensity is always the same at points located "across" the center of the diffraction pattern.

D) We have not used any property of the slit in the derivation of the previous result. So it applies to arbitrary collection of apertures. The only requirement is that the wave must be normally incident. For other than normal incidence, the center of the symmetry moves the same way as in the problem A).

P1:

A) One of these pictures can not be a far-field diffraction pattern. Which? Note that one of the pictures is an actual photograph from:
R.C. Smith and J.S. Marsh, JOSA **64** (1974) 798.

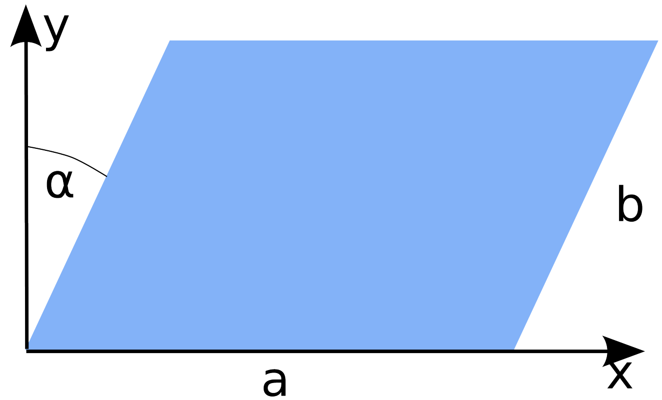




B) What aperture do you think the diffraction pattern corresponds to?

P2: Two dimensional diffraction.

A) Calculate the pattern of diffraction form an aperture in the form of a parallelogram.



B) describe where are zeros of intensity located

Solution:

Set up the diffraction integral (omit un-interesting factors):

$$E \sim \iint_S dx dy e^{-ik(xX+yY)/R}$$

One way to set up the bounds is:

$$E \sim \int_0^{b\cos\alpha} dy \int_{y\tan\alpha}^{y\tan\alpha+a} dx e^{-ik(xX+yY)/R}$$

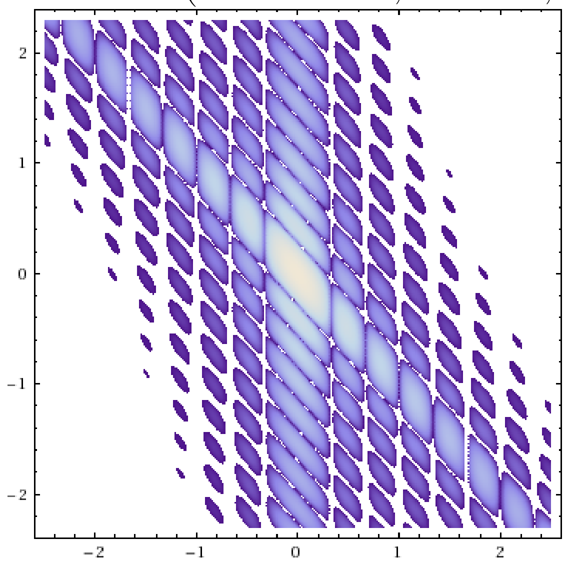
Integration gives:

$$E \sim \frac{R^2}{k^2 X (Y + X \tan\alpha)} \left(-1 + e^{-ikaX/R} + e^{-ib\cos\alpha(Y + X\tan\alpha)} - e^{-ikaX/R} e^{-ib\cos\alpha(Y+X\tan\alpha)} \right)$$

This result is proportional to:

$$E \sim \operatorname{sinc}\left(\frac{akX}{2}\right) \ \operatorname{sinc}\left(\frac{kb\cos\alpha}{2}(Y + X\tan\alpha)\right)$$

Log-scale map of intensity will look like this (for $a = 3\lambda$, $b = 5\lambda$, α =45 deg):



Note: There is two "streaks" each corresponding to a pair of parallel edges, with the streak perpendicular to it. This is a characteristic feature of simple diffraction patterns.