Problem 1:
Consider the plane wave traveling in free space with the wave vector $\vec{k} = 2\pi(1,1,1)/(\sqrt{3}\lambda)$, where $\lambda = 600$nm, and with an electric field amplitude of $E_0 = 30$V/m. The phase of the wave is 2π at the origin (i.e. the magnitude of the electric field vector at $x = 0, t = 0$ is E_0). Direction of the electric field oscillation is given by a unit vector $\hat{e}$.

A) Find an expression for the wave using a real representation. State the condition for $\hat{e}$ that must be satisfied to ensure that divergence of the electric field vanishes.

B) Find an expression for the wave using the complex notation.

C) Find the frequency ν of the wave in THz.

D) Find a point other than the origin where the phase is 2π at $t = 0$

E) Describe (either using words or by giving an equation) the phase fronts of this wave

F) Give two possible choices for $\hat{e}$ (so that divergence of the field is zero)

G) For your first choice in F), specify the unit vector $\hat{b}$ for the direction of oscillation of the magnetic field

H) Calculate the amplitude of the magnetic field, and remember to specify units

I) Give an expression for, and calculate the irradiance

Problem 2:
Consider the plane wave traveling in free space with the wave vector $\vec{k} = 2\pi(1,1,0)/(\sqrt{2}\lambda)$, where $\lambda = 800$nm, and with an electric field amplitude of $E_0 = 3 \times 10^8$V/m. The phase of the wave is $\pi/2$ at the origin (i.e. the magnitude of the electric field vector at $x = 0, t = 0$ is zero). Direction of the electric field oscillation is given by a unit vector $\hat{e}$.

A) Find an expression for the wave using a real representation. State the condition for $\hat{e}$ that must be satisfied to ensure that divergence of the electric field vanishes.

B) Find an expression for the wave using the complex notation.

C) Find the frequency ν of the wave in THz.

D) Find a point other than the origin where the phase is 2π at $t = 0$

E) Describe (either using words or by giving an equation) the phase fronts of this wave

F) Give two possible choices for $\hat{e}$ (so that divergence of the field is zero)

G) For your second choice in F), specify the unit vector $\hat{b}$ for the direction of oscillation of the magnetic field

H) Calculate the amplitude of the magnetic field, and remember to specify units

I) Give an expression for, and calculate the irradiance

Problem 3:
Which of these pairs **can** represent an electromagnatic plane wave propagating in free space? If not, say why. If yes, specify sufficient condition(s) that must be satisfied. Note that such conditions need not be unique — it is OK to choose an easy way out.

A)
$$\vec{E} = \hat{\imath} E_0 \cos[kx - \omega t] \qquad \vec{B} = \hat{\jmath} E_0/c \cos[kx - \omega t]$$

B)
$$\vec{E} = \hat{k} E_0 \cos[kx - \omega t] \qquad \vec{B} = \hat{\jmath} E_0/c \cos[kx - \omega t]$$

C)
$$\vec{E} = \hat{\imath} E_0 \exp[i(kz - \omega t)] \qquad \vec{B} = \hat{\jmath} B_0 \exp[i(kz - \omega t)]$$

D)
$$\vec{E} = \hat{e} E_0 \exp[i(u(z + y + x) - \omega t)] \qquad \vec{B} = \hat{b} B_0 \exp[i(u(x + y + z) - \omega t)]$$

E)
$$\vec{E} = \hat{\jmath} E_0 \cos[kx - \omega t] \qquad \vec{B} = \hat{k} E_0/c \sin[kx - \omega t]$$

F)
$$\vec{E} = \hat{e} E_0 \exp[i\frac{2\pi}{\lambda}(x - y - ct)] \qquad \vec{B} = \hat{b} B_0 \exp[i\frac{2\pi}{\lambda}(x - y - ct)]$$

G)
$$\vec{E} = \hat{k} E_0 \exp[i(\frac{2\pi}{\sqrt{2}\lambda}(x - y - \sqrt{2}ct) + \pi)] \qquad \vec{B} = (\hat{\imath} + \hat{\jmath}) E_0/(\sqrt{2}c) \exp[i\frac{2\pi}{\sqrt{2}\lambda}(x - y - \sqrt{2}ct)]$$

Problem 4:
Given are electric and magnetic fields:

$$\vec{E} = \hat{k} E_0 \exp[i \frac{2\pi}{\sqrt{2}\lambda}(x + y - \sqrt{2}ct)] \qquad \vec{B} = (\hat{i} - \hat{j}) E_0 / (\sqrt{2}c) \exp[i \frac{2\pi}{\sqrt{2}\lambda}(x + y - \sqrt{2}ct)]$$

A) State the Ampere's law (in differential form) for free space

B) Demonstrate by direct calculations that the above fields do satisfy the equation you gave in A)

Problem 5:
Given are electric and magnetic fields:

$$\vec{E} = \hat{k} E_0 \exp[i \frac{2\pi}{\sqrt{2}\lambda}(x + y - \sqrt{2}ct)] \qquad \vec{B} = (\hat{i} - \hat{j}) E_0/(\sqrt{2}c) \exp[i \frac{2\pi}{\sqrt{2}\lambda}(x + y - \sqrt{2}ct)]$$

A) State the Faraday's law for free space

B) Demonstrate by direct calculations that the above fields do satisfy the equation you gave in A)