

Problem 1:

The electric and magnetic fields representing a plane wave (propagating in vacuum) are specified as follows:

$$\vec{E} = \hat{i}E_0 \cos[kz - \omega t] \quad \vec{B} = \hat{j}B_0 \cos[kz - \omega t]$$

with $E_0 = 10^9 \text{ V/m}$ and $\omega = 5 \times 10^{15} \text{ s}^{-1}$.

- A) Identify the wave vector \vec{k} of this plane wave and express it, using the dispersion relation, in terms of ω
- B) Use complex representation plus operator equivalencies, and Ampere's law to show that $B_0 = E_0/c$
- C) Calculate the instantaneous Poynting vector \vec{S} at the origin of coordinate system, $x, y, z = 0$ for a general time t
- D) Calculate the time-averaged Poynting vector $\langle \vec{S} \rangle$ and
- E) What energy corresponds to a single photon in this plane wave?
- E) Calculate the photon flux Φ (i.e. the number of photons crossing a given area per unit of time) over an area of $A = 100 \mu\text{m}^2$

Problem 2:

The electric and magnetic fields representing a plane wave (propagating in vacuum) are specified as follows:

$$\vec{E} = \hat{k} E_0 \sin\left[\frac{2\pi}{\lambda} \hat{n} \cdot \vec{r} - \frac{2\pi c}{\lambda} t\right] \quad \vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} B_0 \sin\left[\frac{2\pi}{\lambda} \hat{n} \cdot \vec{r} - \frac{2\pi c}{\lambda} t\right]$$

with $E_0 = 1 \text{ V/m}$ and $\lambda = 1 \mu\text{m}$ and $\hat{n} = (1, 1, 0)/\sqrt{2}$

A) Explain the meaning of \hat{n}

B) Use complex representation and Faraday's law to show that $cB_0 = E_0$

C) Calculate the instantaneous Poynting vector \vec{S} at the origin of coordinate system, $x, y, z = 0$ for a general time t

D) Calculate the time-averaged Poynting vector $\langle \vec{S} \rangle$ and

E) What energy corresponds to a single photon in this plane wave?

E) Calculate the photon flux Φ (i.e. the number of photons crossing an area per unit of time) over an area of $A = 1 \text{ m}^2$