**Problem 1**

A laser beam is characterized by its power $P = 10^{11}$ W, and maximal irradiance $I_0 = 10^{15}$ W/m$^2$. The wavelength is $\lambda = 1\mu$m.

A) calculate the waist "area" $A$ of the beam

$$P = AI_0 \qquad A = P/I_0 = \frac{10^{11}W}{10^{15}Wm^{-2}} = 10^{-5}m^2$$

B) calculate the Rayleigh range $z_R$

$$A = \frac{\pi w_0^2}{2} = \frac{P}{I_0} \qquad z_R = \frac{\pi w_0^2}{\lambda} = \frac{2P}{I_0\lambda} = \frac{2\times 10^{-4}m^2}{10^{-6}m} = 200m$$

C) calculate the profile of the on-axis intensity $I(z)$ as function of the distance $z$ from the beam waist. Sketch your result in a graph, clearly marking important features and the length scale.

Power in the beam is the same at each $z$, so

$$P = I(z)\text{Area}(z) = I(z)\frac{\pi w(z)^2}{2} = I(z)\frac{\pi}{2}w_0^2\left(1+\left(\frac{z}{z_R}\right)^2\right) = I(z)\frac{P}{I_0}\left(1+\left(\frac{z}{z_R}\right)^2\right)$$

so

$$I(z) = \frac{I_0}{\left(1+\left(\frac{z}{z_R}\right)^2\right)}$$

This curve is "Lorentzian", you should mark $z_R$ where the intensity decreases to one half of the maximum.

D) what is the Full Width at Half Maximum (FWHM) of intensity diameter of the beam? Express it in terms of the $1/e^2$-radius of the intensity profile.

$1/e^2$ radius (of intensity) is $w_0$. Now, FWHM is obtained from:

$$\exp[-\frac{2R^2}{w_0^2}] = \frac{1}{2} \qquad FWHM = 2R = w_0\sqrt{2\ln 2}$$

so $w_0$ is about 85% of FWHM.

E) Where (at what $z$) is the phase front of the beam planar? Where does the phase front have maximal curvature (minimal radius)?

At the waist, and at $\pm z_R$, respectively.

**Problem 2**
A laser beam has power $P = 3$ W, and the wavelength is $\lambda = 1\mu$m.

A) Calculate the photon flux $N$ (photons per second)

$$P = N\hbar\omega = N\hbar\frac{2\pi c}{\lambda} \qquad N = \frac{P\lambda}{2\pi\hbar c}$$

B) Calculate the force exerted by the beam if it is completely absorbed.

Think of a time interval $\Delta t$, and the impulse transferred to the mirror.

$$F\Delta t = \Delta t N\hbar|\vec{k}| = \Delta t\frac{P\lambda}{2\pi\hbar c}\hbar\frac{2\pi}{\lambda} \qquad F = \frac{P}{c} = \frac{3W}{3\times 10^8 ms^{-1}} = 10^{-8}N$$

Note that the results does not contain $\hbar$.

C) Calculate the force when the beam completely reflects at normal incidence

The force is twice that for absorbtion, because *change* in momentum doubles.

$$F = \frac{2P}{c}$$

D) What is the force if the beam reflects (completely) from a mirror at an angle $\alpha = 45$deg?

The normal part of the momentum is $\cos\alpha = 1/\sqrt{2}$ of the whole, but this is doubled because of reflection so we get:

$$F = \frac{P}{c}2\frac{1}{\sqrt{2}}$$

E) What is the force at 45 degrees, if one half of the beam power is absorbed in a "lossy" mirror?

Split, in a thought experiment, the beam into one half that is absorbed, and one half that is completely reflected, add the forces from each (keeping in mind that they are vectors!):

$$\vec{F} = \vec{F}_A + \vec{F}_R = \frac{P/2}{c}\frac{(\hat{i}+\hat{j})}{\sqrt{2}} + \hat{i}\frac{P/2}{c}2\frac{1}{\sqrt{2}}$$

where we have used results of B) and D) in the first and second term respectively.

**Problem 3**
Estimate the force $F$ Sun's radiation exerts on Earth. For simplicity, assume that all radiation that reaches Earth is absorbed. Sun's radiative power is $P = 4 \times 10^{26}$ W, distance from Earth is $D = 1.5 \times 10^{11}$ m, and Earth's mean radius is $R = 6.3 \times 10^{3}$ km.

A) Derive the formula for $F$ from the classical radiation pressure expression, and evaluate.

$$F = \text{pressure} \times \text{area} = \frac{I}{c}\,\pi R^2 = \frac{P}{4\pi D^2\ c}\pi R^2 = \frac{PR^2}{4D^2\ c} \approx 10^8 N$$

B) Derive $F$ from the quantum point of view, assuming that all Sun's radiation is emitted at a single wavelength, and that all photons are absorbed.

see the previous problem — $\hbar$ cancells ...