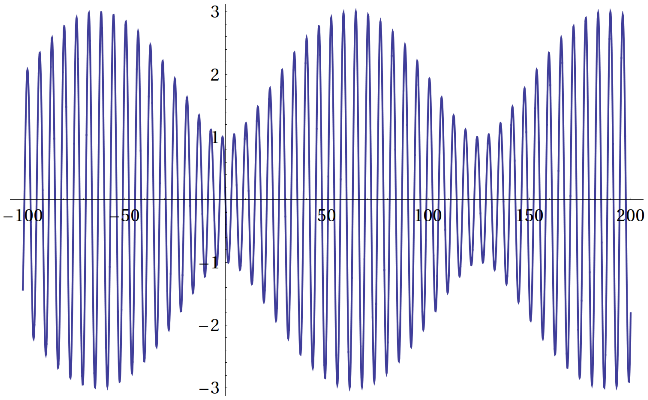
Problem 1:
Two waves interfere and give rise to the following pattern:



A) What can you say about the relation between the two waves propagation numbers k_1 and k_2? Are they "similar" or "rather different?" Explain in words what is the feature you look for to decide your answer.

B) Using the plot, estimate the average propagation number $k_\Sigma = (k_1 + k_2)/2$

so it is likely not far from one.

C) Identify an example of a point where destructive interference occurrs.

D) Identify an example of a point where the interference is contructive.

E) By estimating the resulting amplitudes (at the points identified above) from the given graph, caclulate the approximate amplitude of each wave.

Problem 2:
This problem deals with the propagation of waves in one dimension. Consider a medium in which a scalar wave propagates with the velocity $v = 300m/s$.

A) Write down the wave equation for this medium.

$$\left(\partial_{xx} - \frac{1}{v^2}\partial_{tt}\right)\psi(x,t) = 0$$

B) Explain the notion of the dispersion relation.

C) If the wave has the wavelenth $\lambda = 1m$, what is its temporal frequency?

D) What is the propagation number k?

E) Write down a harmonic-wave solution to the wave equation (for the specified medium) that has a temporal frequency of $\nu = 10^3 Hz$, and propagates along the negative x-axis. Show both, real and complex representations of the wave.

F) By direct calculation, show that for arbitrary differentiable functions $F(u)$ and $G(u)$,

$$F(x - vt) + G(x + vt)$$

is a solution to the wave equation. Which of the two parts propagates in the positive x-axis direction?

H) What is a standing wave? Write down a formula for an example.

Problem 3:
This problem is about 1D waves and complex representation. Consider a medium with a wave propagation speed v. Given is a function:

$$\psi(x,t) = A\cos[kx - \omega t] + A\cos[kx + \omega t - \pi]$$

A) Give a complex representation of this wave

B) Using your answer from A), show that the above solution can be also written as

$$\psi(x,t) = 2A\sin[kx]\sin[\omega t]$$

C) Insert the function given in B) into the wave equation and show by direct calculation that it is a solution provided the propagation number and the angular frequency are related through the dispersion relation.

D) Form your calculations in C) it shoudl be obvious that also

$$\phi(x,t) = -2A\sin[kx + s]\sin[\omega t + \tau]$$

is a solution to WE. Express this as a superposition of two counter-propagating waves. Hint: It is easiest to go through the complex representation.