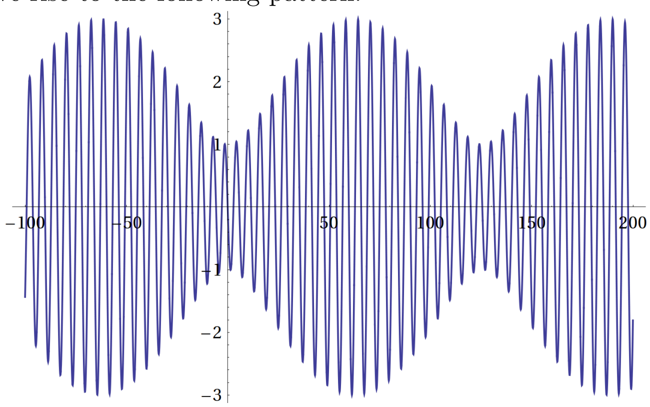
Problem 1:
Two waves interfere and give rise to the following pattern:



A) What can you say about the relation between the two waves propagation numbers k_1 and k_2? Are they "similar" or "rather different?" Explain in words what is the feature you look for to decide your answer.

Similar wavelengths. The telling feature: Long (compared to the wavelength) beat.

B) Using the plot, estimate the average propagation number $k_\Sigma = (k_1 + k_2)/2$

We count about 33 periods between $x = 0$ and $x = 200$, so

$$\lambda \approx 200/33 \approx \frac{2\pi}{k_\Sigma} \qquad k_\Sigma \approx 33\pi/100$$

so it is likely not far from one.

C) Identify an example of a point where destructive interference occurrs.

$$x \approx 0$$

D) Identify an example of a point where the interference is contructive.

$$x \approx 70$$

E) By estimating the resulting amplitudes (at the points identified above) from the given graph, caclulate the approximate amplitude of each wave.

The minimum amplitude occurs when the two amplitudes "subtract":

$$1 \approx A_{big} - A_{small} \ ,$$

while the maximum amplitude occures when they "add:"

$$3 \approx A_{big} + A_{small} \ ,$$

So

$$A_{big} = 2 \quad A_{small} = 1$$

Problem 2:
This problem deals with the propagation of waves in one dimension. Consider a medium in which a scalar wave propagates with the velocity $v = 300m/s$.

A) Write down the wave equation for this medium.

$$\left(\partial_{xx} - \frac{1}{v^2}\partial_{tt}\right)\psi(x,t) = 0$$

B) Explain the notion of the dispersion relation.
It is a relation between the angular frequency and the propagation number, that is required to be satisfied for

$$\text{Any-function-of}(kx - \omega t)$$

to be a solution to WE. In this case it is $k = \omega/v$.

C) If the wave has the wavelenth $\lambda = 1m$, what is its temporal frequency?

$$\nu = \frac{v}{\lambda} = \frac{300ms^{-1}}{1m} = 300Hz$$

(Always show units!)

D) What is the propagation number k?

$$k = \frac{2\pi}{\lambda} = 2\pi m^{-1}$$

E) Write down a harmonic-wave solution to the wave equation (for the specified medium) that has a temporal frequency of $\nu = 10^3 Hz$, and propagates along the negative x-axis. Show both, real and complex representations of the wave.
First find the propagation number from the dispersion relation: $k = \omega/v = 2\pi\nu/c$
real: $A\cos[2\pi\nu x/c - 2\pi\nu t]$ complex: $A\exp[i(2\pi\nu x/c - 2\pi\nu t)]$

F) By direct calculation, show that for arbitrary differentiable functions $F(u)$ and $G(u)$,

$$F(x - vt) + G(x + vt)$$

is a solution to the wave equation. Which of the two parts propagates in the positive x-axis direction?
It is sufficient to do calculation with one of these, as everything repeats:

$$\left(\partial_{xx} - \frac{1}{v^2}\partial_{tt}\right)F(x - vt) = 1^2F''(x - vt) - \frac{1}{v^2}(-v)^2F''(x - vt) = 0$$

Wave propagating in positive x-direction: $F(x - vt)$ (look for opposite signs of x and t).

H) What is a standing wave? Write down a formula for an example.
It is a wave that oscillates with a fixed spatial amplitude profile. Usually it occurs due to interference between waves traveling in opposite directions. E.G.:

$$A\cos[kx]\sin[\omega t]$$

Problem 3:
This problem is about 1D waves and complex representation. Consider a medium with a wave propagation speed v. Given is a function:

$$\psi(x,t) = A\cos[kx - \omega t] + A\cos[kx + \omega t - \pi]$$

A) Give a complex representation of this wave

$$\psi_c(x,t) = A\exp[i(kx - \omega t)] + A\exp[i(kx + \omega t - \pi)]$$

B) Using your answer from A), show that the above solution can be also written as

$$\psi(x,t) = 2A\sin[kx]\sin[\omega t]$$

$$\psi_c(x,t) = A\exp[i(kx-\omega t)]+A\exp[i(kx+\omega t-\pi)] = A\exp[ikx]\left(\exp[-i\omega t] - \exp[+i\omega t]\right) = -2iA\exp[ikx]\sin(\omega t)$$

which has the real part like this:

$$\psi(x,t) = 2A\sin(kx)\sin(\omega t)$$

C) Insert the function given in B) into the wave equation and show by direct calculation that it is a solution provided the propagation number and the angular frequency are related through the dispersion relation.

This is straightforward, but might still be made easier by remembering that *second* derivatives of trig functions leave them invariant plus change their sign. Then you only need to remember that the inner derivative must be done twice. In effect it is yet "another operator equivalence," this time for trig functions.

$$\partial_{xx} 2A\sin(kx)\sin(\omega t) = (-1)k^2 2A\sin(kx)\sin(\omega t) \qquad \partial_{tt} 2A\sin(kx)\sin(\omega t) = (-1)\omega^2 2A\sin(kx)\sin(\omega t)$$

Inserting the above in WE gives you zero *provided* $k^2 = \omega^2/v^2$.

D) Form your calculations in C) it shoudl be obvious that also

$$\phi(x,t) = -2A\sin[kx + s]\sin[\omega t + \tau]$$

is a solution to WE. Express this as a superposition of two counter-propagating waves. Hint: It is easiest to go through the complex representation.

$$\phi(x,t) = -2A\sin[kx+s]\sin[\omega t+\tau] = -2A\frac{\exp[+i(kx+s)] - \exp[-i(kx+s)]}{2i}\,\frac{\exp[+i(\omega t+\tau)] - \exp[-i(\omega t+\tau)]}{2i}$$

now expand and group terms with *same* and *opposite* signs in front of k and/or ω, e.g.

$$-2A(\exp[+i(kx+s)]\exp[+i(\omega t+\tau)] + \exp[-i(kx+s)]\exp[-i(\omega t+\tau)])/(-4)+ \quad (1)$$
$$+2A(\exp[+i(kx+s)]\exp[-i(\omega t+\tau)] + \exp[-i(kx+s)]\exp[+i(\omega t+\tau)])/(-4) \quad (2)$$

... and recognize *cos* in the above expressions to obtain the result:

$$A\cos(kx + \omega t + s + \tau) - A\cos(kx - \omega t + s - \tau)$$

where the first wave propagates left, the second propagates to the right (again: equal signs on x, t mean going left, opposite signs on x, t mean going right).