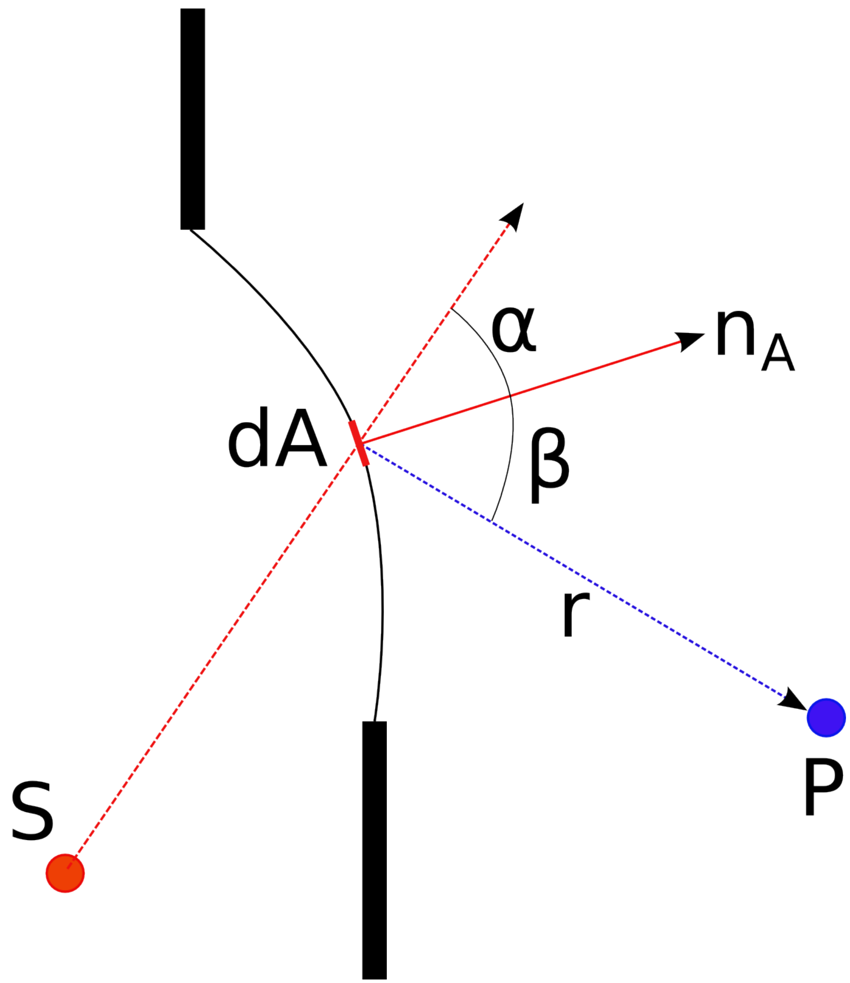
Huygens-Fresnel principle revisited

So far we have used Huygens-Fresnel Principle (HFP) in various forms, approximating its "ingredients" as appropriate for the given situation.

One particular issue not addressed so far is that HFP does not explain why all secondary wavelets only propagate "forward."

Q: Suppose we do not want to commit to any (additional) approximation (perhaps evaluating the diffraction integral numerically). What is then the "exact" mathematical form of HFP?



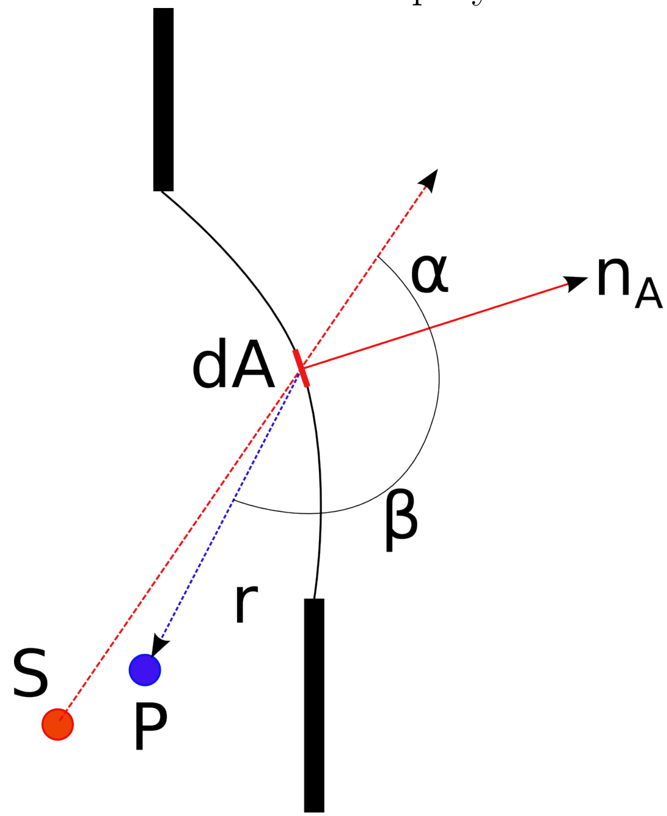
Fresnel-Kirchhof Diffraction:

$$U_{FK} = \frac{-i}{\lambda} \int_{AREA} U_o(x, y, z = 0) \frac{e^{ikr}}{r} \frac{1}{2} [\cos\alpha + \cos\beta] \, dA$$

Role of the obliquity factor

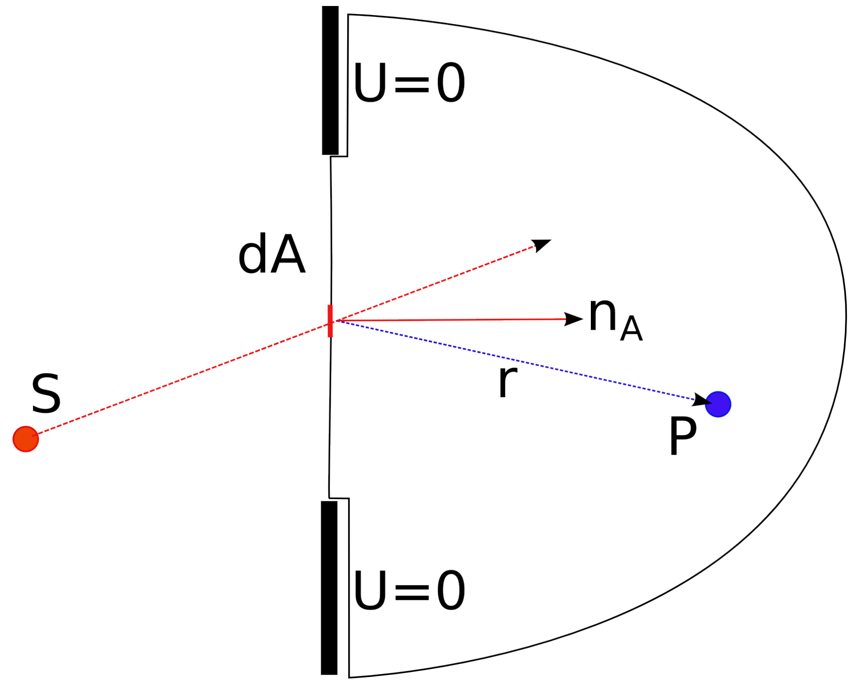
$$\frac{1}{2}(\cos\alpha + \cos\beta)$$

Note: There is different conventions to form the obliquity factor



Thanks to the combination of directional cosines, secondary sources do not radiate in the backward direction.

- The F-K diffraction formula can be derived "from" the Helmholtz equation (see Hecht)
- It requires integration over a *closed* surface, not only over the aperture
- One other part of that surface is the back side of the screen — where the amplitude is assumed to be zero and the contribution to the integral vanishes



- However, also the normal derivative of the field amplitude must be assumed to be zero at the back of the screen.

Fowles: ... assumptions open to considerable debate
Because: the last two assumptions imply that the only possible solution to Helmholtz would vanish everywhere!

Rayleigh-Sommerfeld diffraction integral

It differs from FK in the form of the obliquity factor:

$$U_{RS} = \frac{-i}{\lambda} \int_{AREA} U_o(x, y, z = 0) \frac{e^{ikr}}{r} \cos \beta dA$$

... can be derived in similar was as FK, but requires slightly different assumptions.

Most situations: FK is practically equivalent to RS

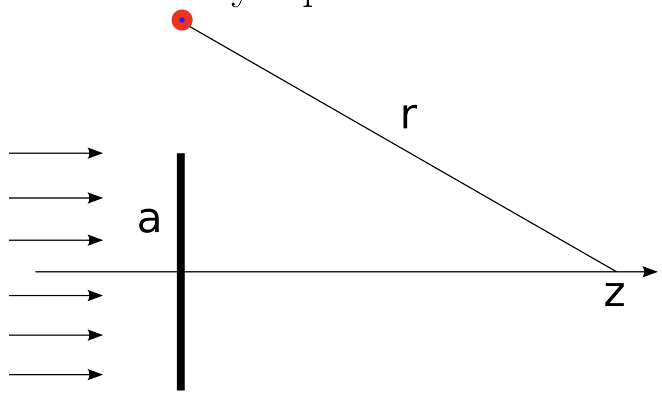
Strongly non-paraxial regime: FK may give non-physical results, that are in contradiction to its underlying assumptions.

Take-away message:

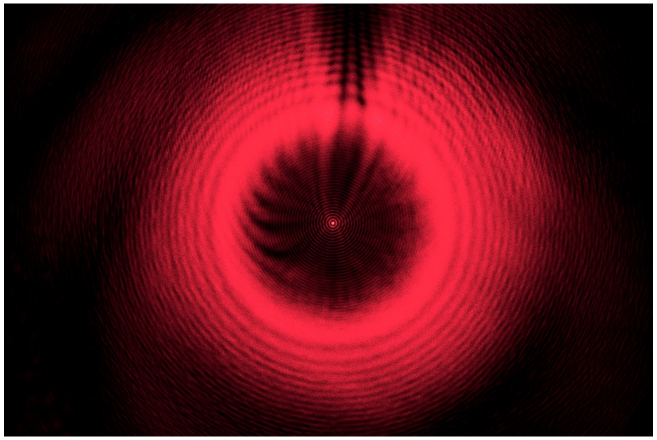
- These difficulties are related to the oversimplified model of the screen. Without specification of its (material and geometric) properties, one does not know what boundary conditions to impose on Helmholtz solutions.
- Diffraction integrals can be used in "benign regimes" (e.g. light propagation from one resonator mirror to the other mirror). Beam-propagation methods, or direct Maxwell solutions must be applied to "more extreme situations."

Poisson bright spot

Diffraction on a circular screen illuminated by a point source from a distance, or by a plane wave.



- Nice illustration of the wave nature of light
- Includes a regime in which propagation is strongly non-paraxial
- Admits solutions: FK, RS, exact vectorial (for a thin PEC screen), BPM, ...



Poisson bright spot: Calculation of on-axis intensity

$$U(0,0,z) = \frac{-i}{\lambda} U_0 \int_{\rho=a}^{\rho\to\infty} \frac{e^{ikr}}{r} \frac{1}{2} \left[c_1 + c_2 \frac{z}{r} \right] 2\pi\rho d\rho$$

Two in one:

$$c_1 = 0 \quad , \quad c_2 = 2 \quad \text{for RS}$$

$$c_1 = 1 \quad , \quad c_2 = 1 \quad \text{for FK}$$

Results (assuming K is large, and using the same trick as for diffraction from a straight edge):

$$U_{RS} = U_0 \frac{z}{r_0} e^{ikr_0} \qquad\qquad U_{FK} = U_0 \frac{1}{2} \left(1 + \frac{z}{r_0} \right) e^{ikr_0}$$

- these results are obviously different
- FK formula is inconsistent with its own assumption: It predicts non-zero amplitude in the center of the screen (immediately behind it). Yet FK integral is derived on the assumtion that it the amplitude iz zero everywhere the screen is blocking the light.

Poisson bright spot: Calculation of on-axis intensity, FK vs RS

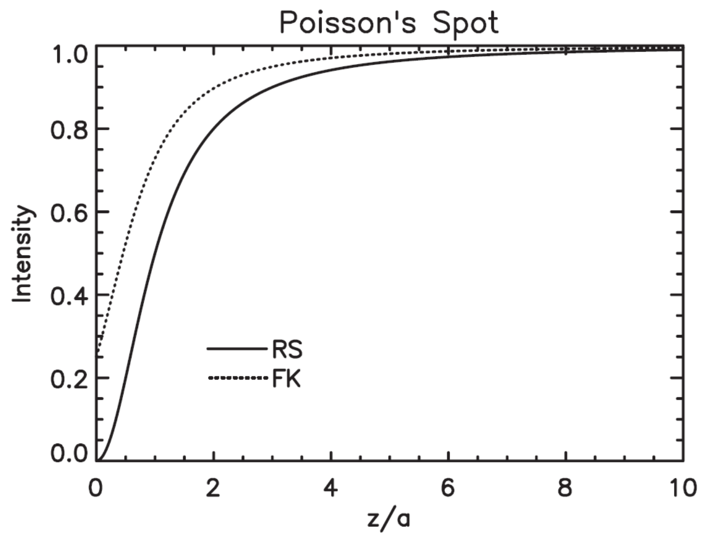
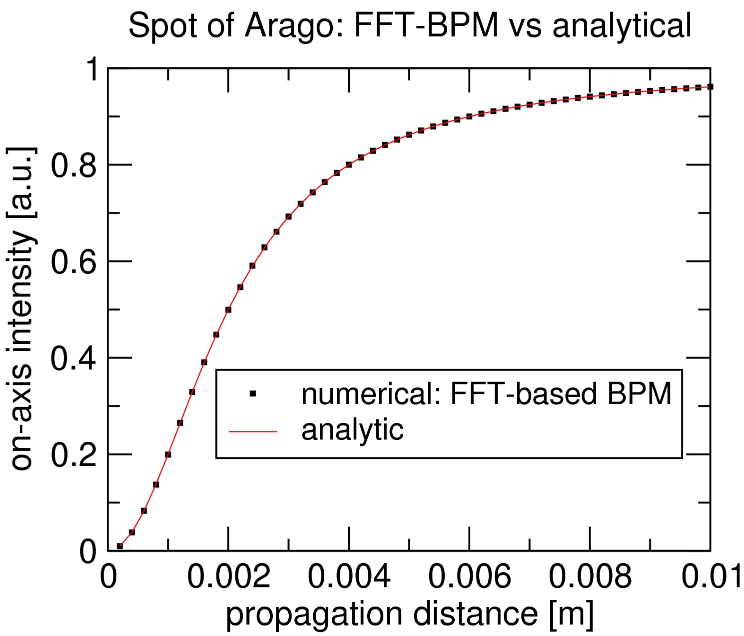


Fig. 3 — Intensity of Poisson's spot at on-axis distance z behind a disk of radius a, according to the Rayleigh-Sommerfeld and Fresnel-Kirchhoff diffraction theories.



Take away:

Poisson bright spot = situation in which FK diffraction integral fails

RS diffraction integral gives the correct result

Exercise: On Poisson bright spot. The intuitive picture of how the central bright spot is generated is based on the notion of Fresnel zones. Imagine that the obstacle (screen) is not a perfect circle, but has a fine-scale, random "perturbations of the local radius."

A) What is the effect of such a "blurry" edge on the intensity of the Poisson spot?

B) Estimate how big must the irregularity of the edge be in order to suppress the spot completely?

C) What happens if we have a slightly elliptical screen (instead of a pefect circle)

D) Do you expect that a simlar bright spot occurs in the one-dimensional diffraction, say on an opaque infinite strip?

E) Where, in relation to the circular screen, do you expect the light propagation to be non-paraxial? Can you intuit how the vectorial nature of light manifests in the intensity profile close to the axis?

F) In the near-axis region, the intensity profile is one of a Bessel beam:

$$I(r) \approx J_0^2\left(\frac{2\pi}{\lambda}\frac{a}{z}r\right)$$

Can you explain why it happens that way?