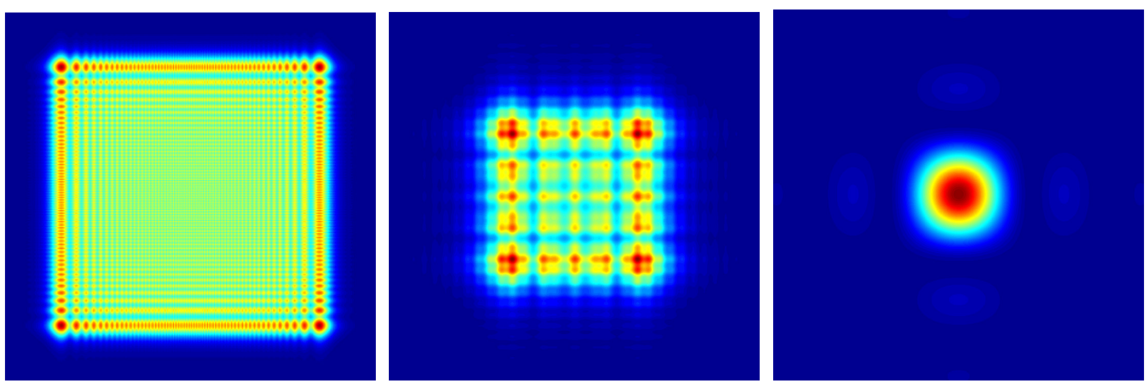
Fresnel vs Fraunhofer diffraction

- Near vs far-field observation
- Need for next-order of accuracy in approximating exponential phase factors,
- More difficult to treat mathematically
- Theory isn't even formulated completely free of internal "conflicts."
  (Example: Different scalar diffraction formulas)
- More intuitive diffraction pattern resemble the aperture shape
- Controls the "boundary between light and shadow"

Fresnel $\rightarrow$ Fraunhofer



Fresnel diffraction: qualitative picture

Fresnel zone:

A portion of an aperture over which the phase, as seen at the point of observation, does not change by more than π. In other words, the amplitude of the diffracted field has roughly the same direction from all wavelet sources within the same zone.

Let R be the radius of the zone, illuminated by a source from a distance h' and observed at distance h beyond the screen, the total traveled path is

$$r + r' = \sqrt{h^2 + R^2} + \sqrt{h'^2 + R^2} \approx h + h' + \frac{1}{2}R^2\left(\frac{1}{h} + \frac{1}{h'}\right) + \dots$$

Divide into regions (circles of $R = \text{const}$) so that $r + r'$ differs by $\lambda/2$ from one boundary to the other:

$$\begin{aligned} R_1 &= \sqrt{1\ \lambda L} \\ R_2 &= \sqrt{2\ \lambda L} \\ R_3 &= \sqrt{3\ \lambda L} \\ R_4 &= \dots \end{aligned}$$

where

$$L = \left(\frac{1}{h} + \frac{1}{h'}\right)^{-1}$$

Fresnel zone **areas are equal:**

$$S_n = \pi R_{n+1}^2 - \pi R_n^2 = \lambda L$$

Typical size: For observation distances on the order of meters, wavelength of a fraction of a micron, R becomes a fraction of a millimeter.

Optical amplitude caused by a finite number of (complete) Fresnel zones:

$$|U_p| = |U_1| - |U_2| + |U_3| - |U_4| + \dots$$

This is roughly zero for even number of zones, and $|U_1|$ for odd number of zones

Refine the above argument. Consider:

- increasing radial distance
- increasing angle of illumination and observation
- both cause $|U_n|$ to slowly decrease with n

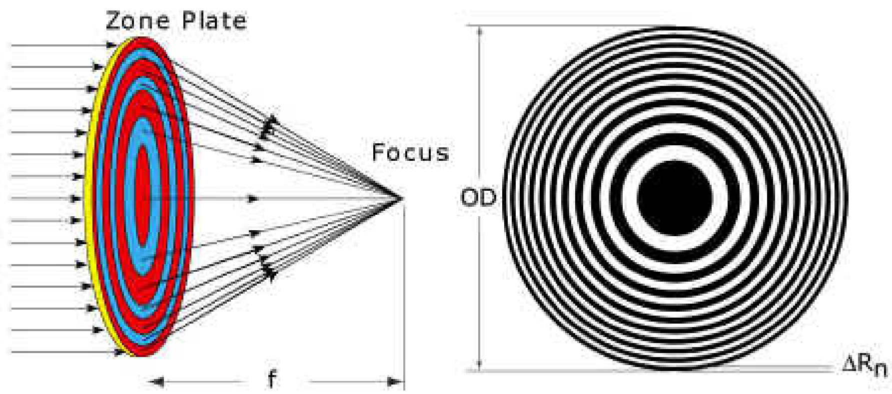
$$|U_p| = \frac{1}{2}|U_1| + (\frac{1}{2}|U_1| - |U_2| + \frac{1}{2}|U_3|) + (\frac{1}{2}|U_3| - |U_4| + \frac{1}{2}|U_5|) + \dots$$

... terms such as $(\frac{1}{2}|U_3| - |U_4| + \frac{1}{2}|U_5|)$ nearly vanish. So, the on-axis illumination is given by the first term.

Bright spot of Arago: On-axis intensity behind a circular obstacle is nearly the same as the illumination without aperture.

Zone plate:

Idea = eliminate every other Fresnel zone. The rest add-up constructively, and act as a lens.



Exercise: In our derivation of the Fresnel zone radius, higher-order terms were neglected, resulting in an approximate expression for R_n. Derive the formula for the radius of the n-th zone of a zone-plate with a focal length f, this time without approximations, and show that it is:

$$r_n = \sqrt{n\lambda f + \frac{n^2\lambda^2}{4}}$$

Also note that this reduces to our previous formula, as it should.