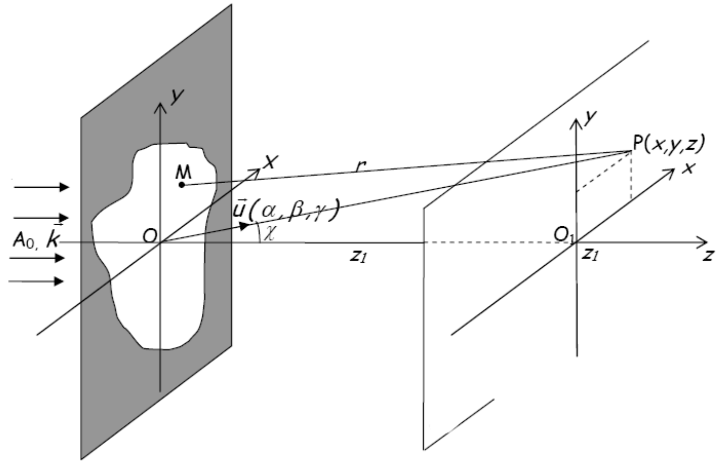
A typical diffraction problem



- Rayleigh-Sommerfeld Diffraction

$$U_{RS} = \frac{-i}{\lambda} \int_{AREA} U_o(x, y, z = 0) \frac{e^{ikr}}{r} \cos(\alpha) dxdy$$

- Fresnel-Kirchhof Diffraction

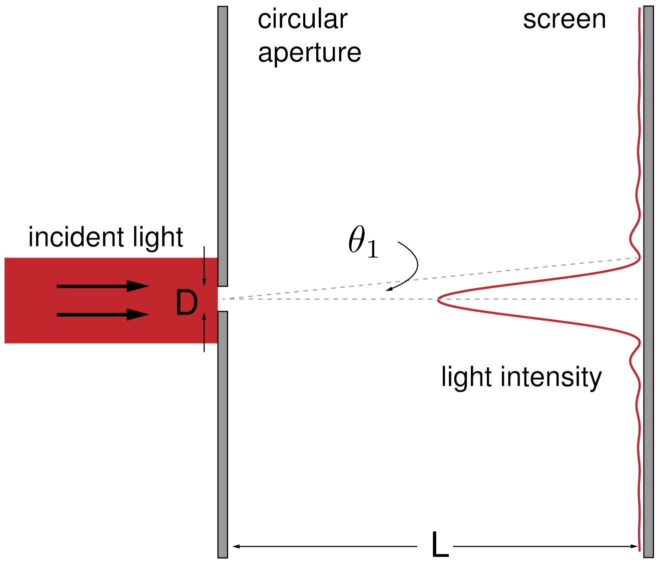
$$U_{FK} = \frac{-i}{\lambda} \int_{AREA} U_o(x, y, z = 0) \frac{e^{ikr}}{r} \frac{1}{2} [\cos\alpha + \cos\beta] \, dxdy$$

- Vectorial Diffraction

$$\vec{E} = \frac{1}{2\pi} \nabla \times \int_{AREA} \vec{n} \times \vec{E} \frac{e^{ikr}}{r} dS$$

We are going to look at simple (but most important) cases...

Diffraction: (according to Sommerfeld) Diffraction is any deviation of light rays from rectilinear paths which cannot be interpreted as reflection or refraction.



Huygens-Fresnel Principle

Huygens 1678: wave theory — ignored in 18th century (Newton's influence)

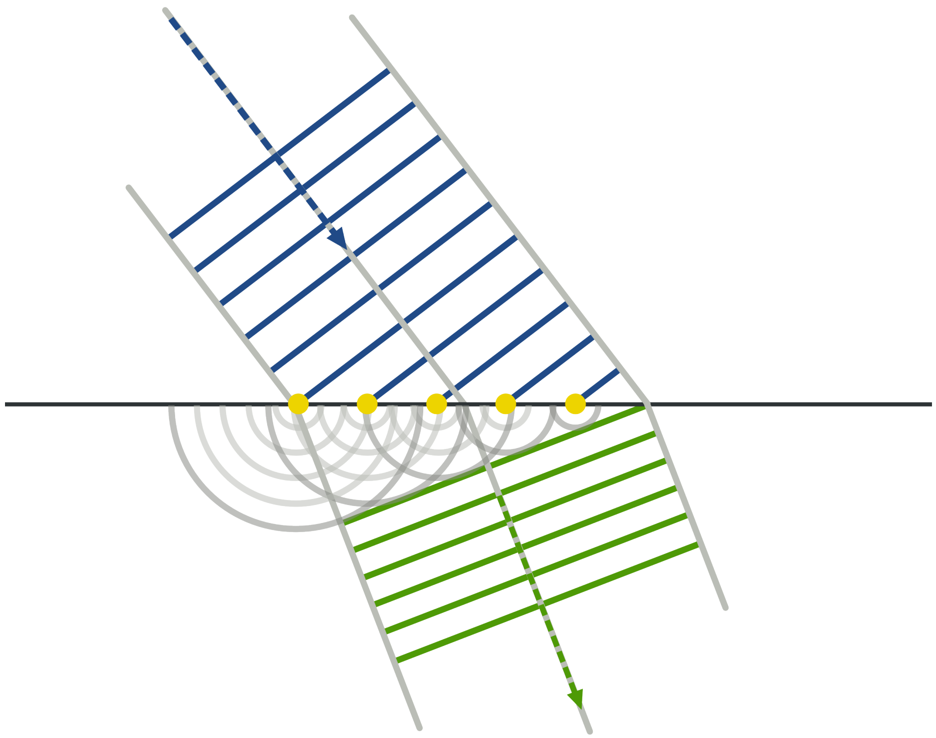
Young 1804: revived the idea of interference

Fresnel 1818: synthesized Huygens & Young into unified wave theory of light

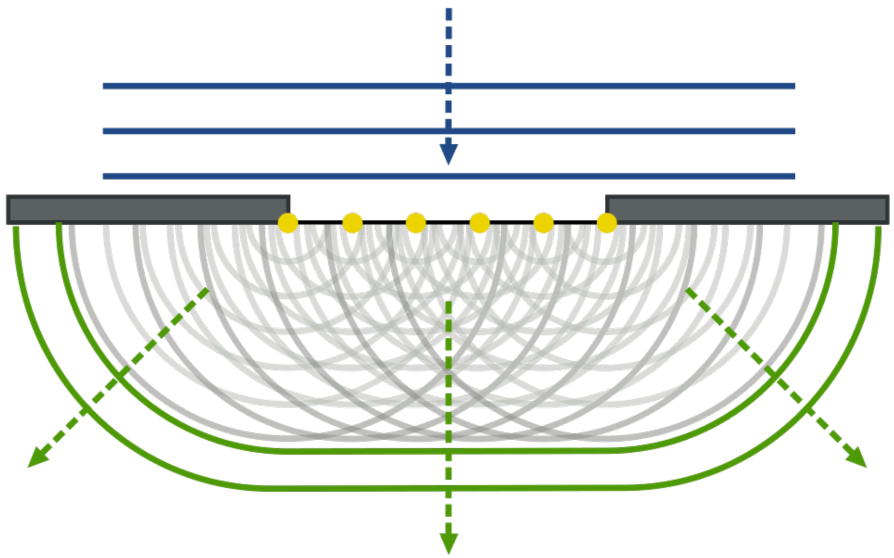
Maxwell 1860— rigorous mathematical basis

HFP: Each point on a wavefront acts as a secondary source of spherical waves. As such, diffraction = interference between secondary sources...

Examples: Refraction viewed as HFP in action:



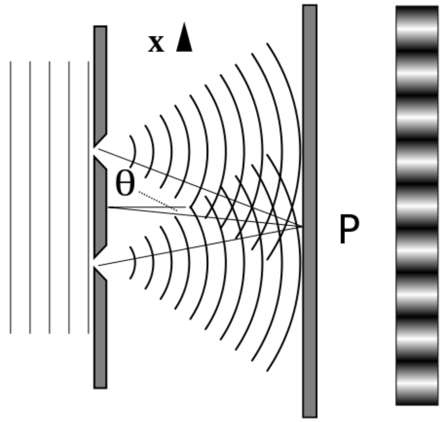
Examples: Transmission through an aperture as HFP:



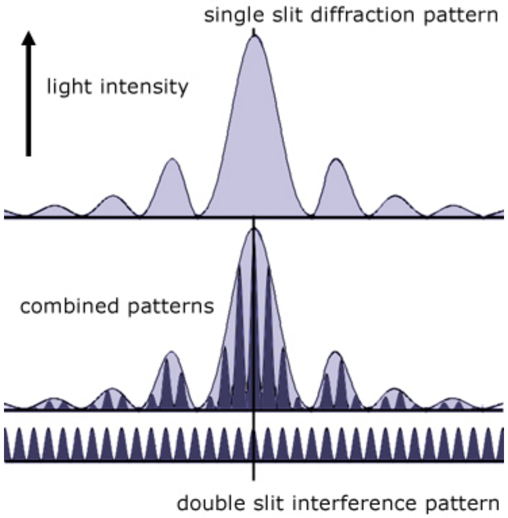
Q: Why no backward propagation? Only solved by Fresnel → obliquity factors

Examples: Young double-slit, with "realistic" apertures

Ideal: Two point sources.

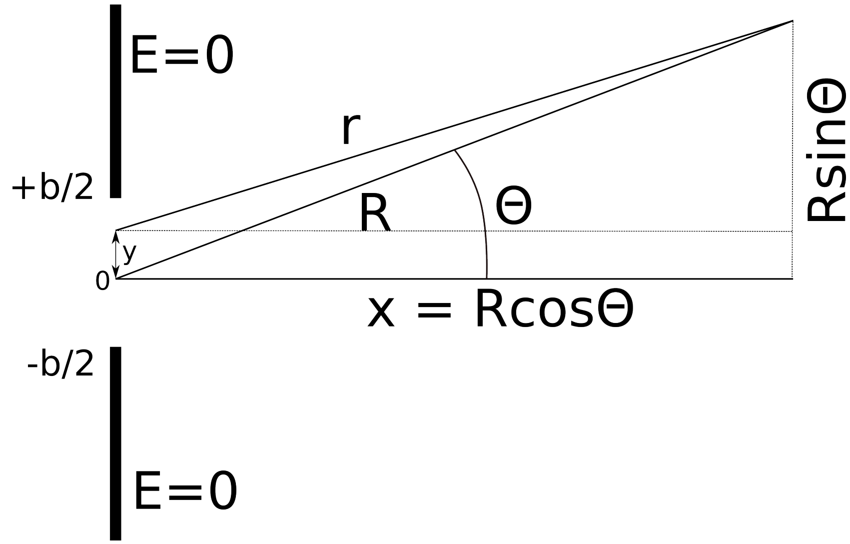


Finite slit width: "Superposition of patterns"



Fraunhofer and Fresnel Diffraction

Consider a single slit geometry:



For each point labeled $-b/2 < y < b/2$ we have a HFP source of equal strength. Approximate the wave emanating from these sources by an outgoing spherical wave:

$$\vec{E}(\vec{r},\omega) = \hat{e}\frac{A}{r}e^{ikr}$$

1. When we assume $\hat{e}$ is parallel to the (infinite) slit, we have effectively scalar problem.
2. We can put $A = 1$

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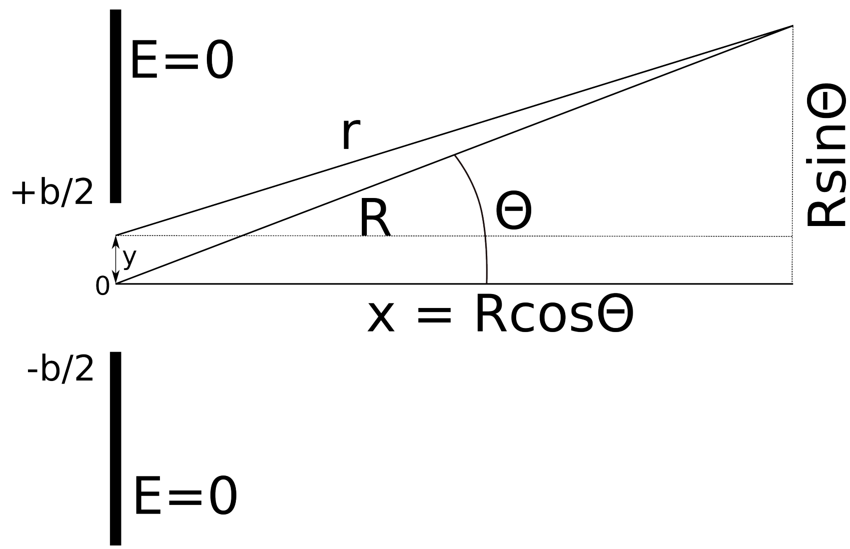

Then for each of the sources:

$$\vec{E}(\vec{r},\omega) = \hat{e}\frac{A}{r}e^{ikr}$$

$$r^2 = x^2 + (R\sin\theta - y)^2 = R^2\cos^2\theta + (R^2\sin^2\theta - 2yR\sin\theta + y^2)$$

giving

$$r = \sqrt{R^2 - (2yR\sin\theta - y^2)} = R\sqrt{1 - \frac{(2yR\sin\theta - y^2)}{R^2}}$$

For a screen very far:

$$r = R\sqrt{1 - \frac{(2yR\sin\theta - y^2)}{R^2}} \approx R\left(1 - \frac{1}{2}\frac{(2yR\sin\theta - y^2)}{R^2}\right)$$

$$r \approx R - y\sin\theta + \frac{1}{2}\frac{y^2}{R} + \dots$$

Fresnel theory: Full form of r or at least linear and quadratic variation. Diffraction effects depend strongly on the distance.

Fraunhofer theory: Keep only the linear term in y

$$R \approx R - y\sin\theta$$

Results in correction to the phase in e^{ikr}. Asking that the Fraunhofer term is smaller than needed to change the sign means:

$$\left(\frac{2\pi}{\lambda}\right)\frac{1}{2}\frac{y^2}{R} < \pi$$

Consider maximal y, i.e. $y = \pm b/2$. The the condition becomes:

$$\frac{b^2}{4\lambda R} < 1$$

or

$$N_F = \frac{(b/2)^2}{\lambda R} < 1$$

The rule of thumb:

- Fresnel region

$$R < \frac{(b/2)^2}{\lambda}$$

- Fraunhofer diffraction

$$R > \frac{(b/2)^2}{\lambda}$$

In a general situation: b is some characteristic dimension of the aperture.