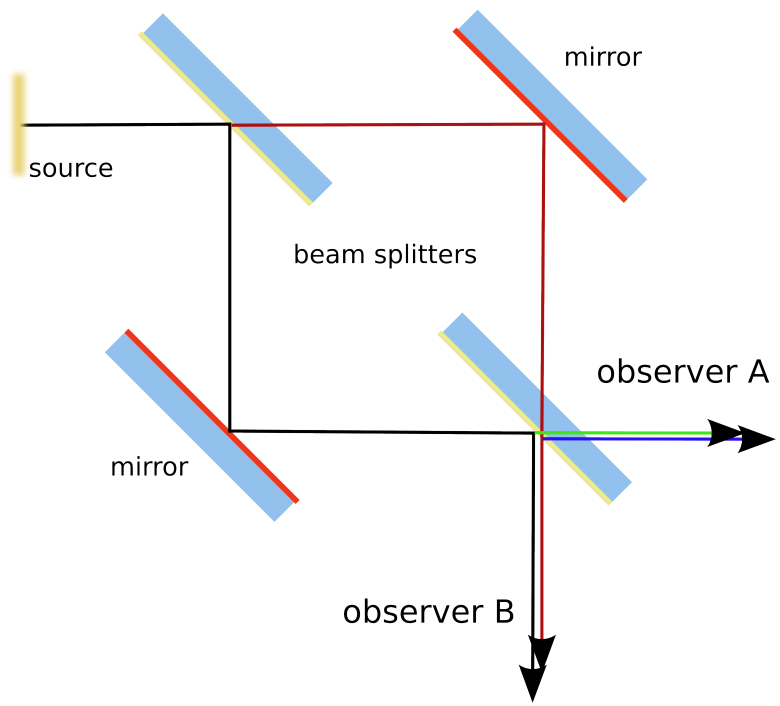
P1: Mach-Zehnder interferometer. In the interferometer shown, 90 percent of light is transmitted by both beam splitters. They both reflect 10 percent (no losses are considered). One of the mirrors is not parallel to the other by an angle δ.



A) What pattern does observer A see?

B) What pattern does observer B see?

Note:

Solution:

Of course, we do not know what pattern exactly can each observer see. Yet we can say something about the contrast in each case, because that is only dependent on the parameters of the interferometer. Figure out the four possible paths and corresponding amplitudes (enter t for transmission through and r for reflection from a beam splitter):

$$E_1 = E_0 t r' e^{ikL_u} \qquad E_2 = E_0 r t e^{ikL_b} \qquad E_3 = E_0 t t' e^{ikL_u} \qquad E_4 = E_0 r r e^{ikL_b}$$

Observer amplitudes:

$$E_A = E_1 + E_2 \qquad E_B = E_3 + E_4$$

Here we can already make a qualitative conclusion: There must be full contrast in the pattern observed by A, because the contributing waves $E_{1,2}$ have the same amplitude. On the other hand, B sees waves with rather different amplitudes which can not "destruct" each other through interference. Therefore the contrast in the pattern seen by B will be (much) less.

The following calculation estimates the difference...

A) Observer A sees interference of equal-intensity beams. A's fringe pattern has therefore full contrast.

$$I_A = |E_A|^2 = (E_1 + E_2)(E_1^* + E_2^*)$$

$$I_A = 4I_0|t|^2|r|^2 \cos^2[k(L_u - L_b)/2] = 4I_0 TR \cos^2[k(L_u - L_b)/2]$$

The "amplitude" of the fringe pattern will be $4I_0TR \approx 0.36I_0$, and the average intensity (constant background) is $2I_0TR$.

B) Observer B sees interference of un-equal intensity beam. The pattern in this case has lower contrast and dark fringes are not completely dark.

$$I_B = (E_3 + E_4)(E_3^* + E_4^*) = |E_3|^2 + E_3E_4^* + E_4E_3^* + |E_4|^2$$

The constant background is given by the first and last term:

$$I_0(T^2 + R^2) \approx 0.8I_0$$

The interference term is

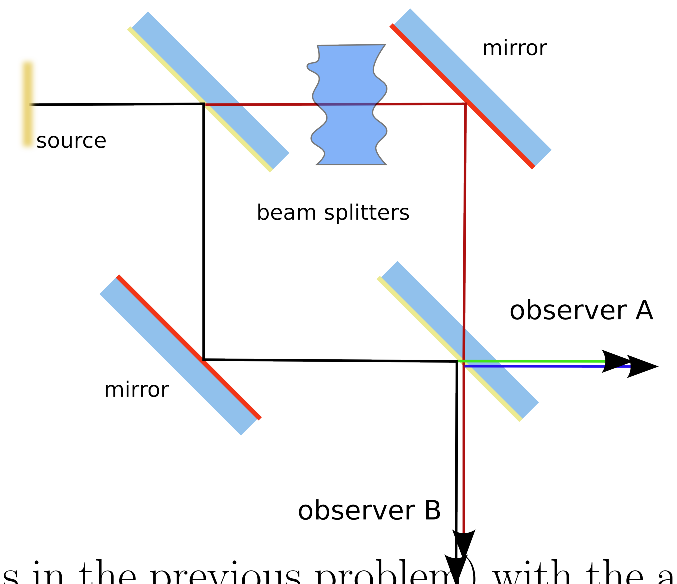
$$2\mathrm{Re}\{E_3E_4^*\} = 2I_0\mathrm{Re}\{tt'r^*r^*e^{i(L_u - L_b)}\}$$

with its "amplitude" proportional to $I_0TR \approx 0.1I_0$

Sanity check: The terms that represent the constant background illumination in A and B add up to I_0 as they should.

P2: Mach-Zehnder interferometer as a switch. Now consider a symmetric MZI, with 50% reflectivity at each beam-splitter. In one of the arms, optical path is modified by an amount of $\delta = \Delta n d$. Calculate the intensity as a function of δ observed by observer A and B. Assume that the reflection and transmission coefficients are given by:

$$r = r' = 1/\sqrt{2} \qquad t = t' = i/\sqrt{2}$$



Solution:

We have the same amplitudes (as in the previous problem) with the addition of a phase-change in one of the paths:

$$E_1 = E_0 t r' e^{ikL_u} e^{i\delta} \qquad E_2 = E_0 r t e^{ikL_b} \qquad E_3 = E_0 t t' e^{ikL_u} e^{i\delta} \qquad E_4 = E_0 r r e^{ikL_b} \ ,$$

but this time $|t|^2 = |r|^2 = 1/2$.

$$E_A = E_1 + E_2 = E_0 t r' e^{ikL_u} + E_0 r t e^{ikL_b} \qquad E_B = E_3 + E_4 = E_0 t t' e^{ikL_u} + E_0 r r e^{ikL_b}$$

$$I_A = |E_A|^2 = I_0 \left(\frac{1}{2} - \frac{1}{2} \cos[\delta + k(L_u - L_b)] \right)$$

$$I_B = |E_B|^2 = I_0 \left(\frac{1}{2} + \frac{1}{2} \cos[\delta + k(L_u - L_b)] \right)$$

- Sum of powers in two output ports is equal to the input
- A and B are complementary in power/intensity variation
- variation of δ can be used to switch power from one output port to the other
- phase change of π needed for complete "on/off"
- smaller variation (of phase) still useful: modulation, sensing, ...
- MZI topology often realized on chips for switching, de-multiplexing, refractive index measurement, ...