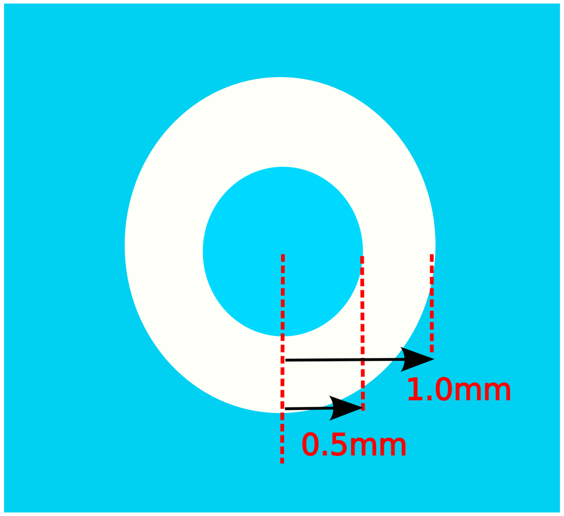
P: Fresnel diffraction. This screen is illuminated by a point source from a distance of 1 m. Calculate the irradiance on-axis 1 m behind the screen. Express it in terms of intensity without the screen. The wavelength is 500 nm.



Solution:
In this kind of problems the key is to determine which Fresnel zones are open, or what fractions of them are open.
Here we have the effective distance

$$L = \left(\frac{1}{h'} + \frac{1}{h}\right)^{-1} = 1/(1+1)\ \mathrm{m} = 0.5\ \mathrm{m}$$

Use this to determine the radia of Fresnel zones:

$$R_1 = \sqrt{1\lambda L} = 0.5\ \mathrm{mm}$$

This means that exactly the First zone is closed by the circle in the center.

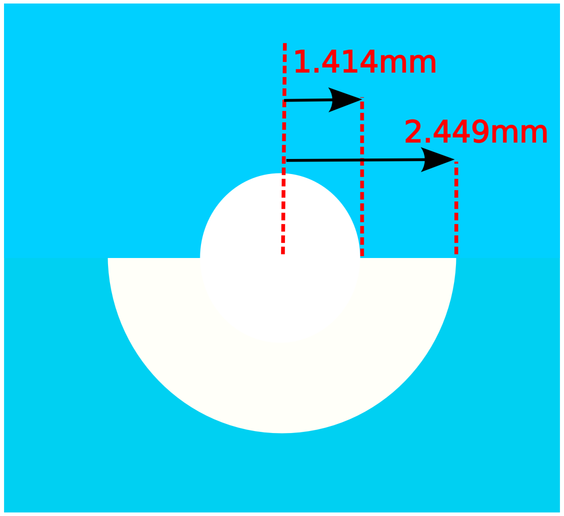
$$R_2 = \sqrt{2\lambda L} = 0.5\sqrt{2}\ \mathrm{mm}$$

$$R_3 = \sqrt{3\lambda L} = 0.5\sqrt{3}\ \mathrm{mm}$$

$$R_4 = \sqrt{4\lambda L} = 0.5\sqrt{4}\ \mathrm{mm} = 1\ \mathrm{mm}$$

This means that exactly three zones are open. Since an odd number of zones is open, the illumination is the same as from a single zone. Amplitude from the first zone is twice the amplitude of unobstructed illumination. So the intensity in this case is four times that as without the screen.

P: Fresnel diffraction. This screen is illuminated by a point source from a very large distance. Calculate the irradiance on-axis 4 m behind the screen. Express it in terms of intensity without the screen. The wavelength is 500 nm. The incident irradiance is 25 W/m^2



Solution:
Because the illumination is by a plane wave $L = 4$ m. Radia of Fresnel zones are

$$R_1 = \sqrt{1\lambda L} = 1.414 \text{ mm}$$

$$R_2 = \sqrt{2\lambda L} = 2.0 \text{ mm}$$

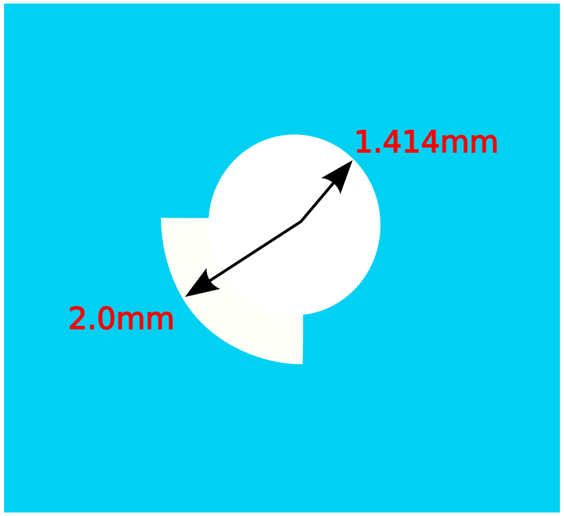
$$R_3 = \sqrt{3\lambda L} = 2.449 \text{ mm}$$

So the first zone is completely open, while the rest of the opening contains one half of each zone two and three.
Use these fractions to modify the equation for the amplitude generated by a finite number of Fresnel zones:

$$U_p = U_1 - 0.5U_2 + 0.5U_3$$

where we know that U_i are approximately equal. The last two terms therefore cancel, and the illumination if effectively only due to the first zone. In the previous problem we have established that a single zone produces four times of the unobstructed intensity. The intensity at the observation point is therefore 100 W/m^2.

P: Fresnel diffraction. This screen is illuminated by a collimated beam with intensity of 40 W/m^2. Calculate the irradiance on-axis 4 m behind the screen. Express it in terms of intensity without the screen. The wavelength is 500 nm. The incident irradiance is 25 W/m^2.



Solution:
Because the illumination is by a plane wave $L = 4$ m. Radia of Fresnel zones are

$$R_1 = \sqrt{1\lambda L} = 1.414 \text{ mm}$$

$$R_2 = \sqrt{2\lambda L} = 2.0 \text{ mm}$$

So the first zone is completely open, while the second is 1/4 opening.
Use these fractions to modify the equation for the amplitude generated by a finite number of Fresnel zones:

$$U_p = U_1 - 0.25U_2 = 3/4U_1 = 3/42U_{unobst}$$

$$I_p = (3/2)^2 I_0 \sim 56 \text{ W/m}^2$$

Practical summary of Fresnel-zones type problems:

Note: I am leaving out absolute value signs — every amplitude should be understood as $|U|$.

- Determine L
- Figure out radius for several zones
- Compare them to the dimensions found in the aperture
- Identify which zones are at least partially open
- If a zone U_i is not full, estimate what fraction f_i of it is open.
- Use the modified formula
$$U_p = f_1 U_1 - f_2 U_2 + f_3 U_3 - \dots$$
- Recall the approximation of equal illumination from each zone:
$$U_1 = U_2 = U_3 = 2U_0$$
where U_0 is the incident (unobstructed) amplitude. Here we are using the result from the class
$$U_0 = \frac{1}{2}U_1 + \sum_{j=2,4,\ldots} (U_{j-1}/2 - U_j + U_{j+1}/2) \approx \frac{U_1}{2}$$
- Observed irradiance scales as $|U_p|^2$, express it in terms of $|U_0|^2 = I_0$

P: A circular aperture is illuminated by a distant source, and irradiance is observed by an on-axis detector that approaches the aperture from an infinite distance. Given is the light wavelength and the radius of the aperture.

A) Describe, qualitatively, what is the observed irradiance as a function of the aperture-detector distance.

B) Find the location of the first few maxima.