Name:	

OPTI 310, Fall 2016 **Final Exam** Prof. M. Kolesik

Dec. 14, 2016.
10:30 am - 12:30 pm

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer ALL questions. Show supporting arguments — unjustified answers receive reduced credit!

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8$ m/s $\quad\quad \hbar = 1.05 \times 10^{-34}$ Js

$\frac{\pi w_0^2}{\lambda} \quad\quad \frac{\pi w_0^2}{2}$

$\frac{1}{2}\epsilon_0 c n E_0^2$

$\sqrt{1\lambda L} \quad \sqrt{2\lambda L} \quad \sqrt{3\lambda L}\ldots \quad L = \left(\frac{1}{h} + \frac{1}{h'}\right)^{-1}$

$\nu = \frac{c}{\lambda} = N\nu_{FSR} \quad\quad \mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} \quad\quad \frac{c}{2d} \quad\quad RP = N\mathcal{F}$

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 \quad , \quad \beta = \frac{\pi w \sin\Theta}{\lambda} \quad , \quad \gamma = \frac{\pi d \sin\Theta}{\lambda}$$

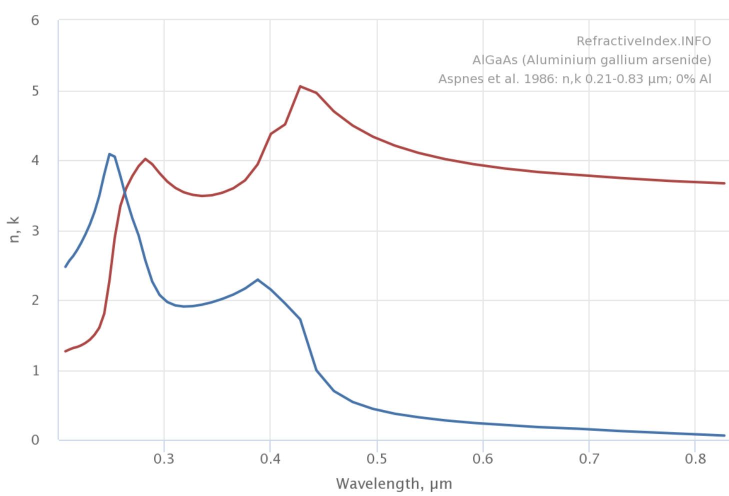
$$U_p \sim |U_1|/2 + (|U_1|/2 - |U_2| + |U_3|/2) + (|U_3|/2 - |U_4| + |U_5|/2) + \ldots \sim |U_1|/2$$

Score:	/out of 60 possible points

1. (20pts) This problem covers a variety of topics from the class. No long calculations are required.

(a - 3pts) An electromagnetic plane-wave propagates in vacuum, and its wave-vector is $\vec{k} = \frac{2\pi}{\sqrt{2}\lambda}(\hat{k} + \hat{\jmath})$. Assuming that the wave polarization is linear, find at least one possible pair of electric and magnetic unit vectors $\hat{e}$ and $\hat{b}$ that give the oscillation direction of their respective fields. Say in which order the three vectors constitute a right-hand oriented system?

(b - 4pts) This figure shows real or imaginary part of index of refraction of Gallium arsenide (GaAs)



1) Which curve shows the real and which the imaginary part of the refractive index?
2) Identify a region in which this material exhibits *significant* absorption.
3) Identify a region in which this material exhibits normal dispersion.
4) Identify in this picture what we call the visible region.

(c - 2pts) Write down (explicitly, not using symbol Δ) a three-dimensional scalar wave equation for propagation with wave velocity u. Give an example of a physical process that can be described by such an equation.

(d - 2pts) The following formula gives an exact solution to the three dimensional scalar wave equation:

$$\psi(r,t) = \frac{1}{r}\exp[i(\frac{2\pi}{\lambda}r - \omega t)]$$

1) Explain how is the factor $1/r$ related to the law of energy conservation.
2) If the wave velocity is c, give the dispersion relation that must be satisfied between λ and ω.

(e - 2pts) Consider a vector field, closely related to the previous spherical wave solution, given like so:

$$\vec{V}(r,t) = \hat{i}\frac{1}{r}\exp[i(\frac{2\pi}{\lambda}r - \omega t)]$$

Is it possible that this vector field represents the electric field of a spherical electromagnetic wave? Hint: Consider the fact that electromagnetic waves are transverse waves. You can use calculations or, (better and shorter) give an argument based on the local propagation direction.

(f - 2pts) Solar radiation has an irradiance of roughly 1.4kW per meter squared (on Earth). If we assume that all photons have wavelength $\lambda = 500$nm, how many photons cross the area of one meter squared per one second?

(g - 3pts) Use the notion of the Fresnel zone to explain the Poisson's bright spot.

(h - 2pts) Describe what a half-wave plate does to the polarization state of a linearly polarized beam in two cases:
A) the oscillation direction of the incident light is aligned with the fast or slow axis of the plate
B) the oscillation direction makes 45 degree angle with the axes of the plate

2. (10 pts) In this problem, consider electromagnetic field as described by Maxwell equations in free space.
(a - 2pts) Finish the equation:

$$-\partial_t B_x = \ldots$$

(b - 2pts) Finish the equation:

$$+\partial_t E_y = \ldots$$

In the following, consider the following plane-wave solution in complex representation for the electric field

$$\vec{E}(\vec{r},t) = \hat{\jmath} E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\pi/2)},$$

where ω is the field angular frequency.

(c - 1pt) Determine $\vec{k}$ in terms of the angular frequency so that the above wave propagates along positive z-axis.

(d - 1pt) Specify the real electric field corresponding to the above complex representation

(e -2pts) Calculate the real magnetic field corresponding to the electric field from (d).

(f - 1pt) Give the *general* (in terms of $\vec{E}$ and $\vec{H}$ or $\vec{B}$) formula for the Poynting vector, and explain its physical meaning.

(g - 1pt) Calculate the time-dependent Poynting vector from the real fields obtained above

3. (10pts) The formulae for the reflection coefficients for s-polarized and p-polarized incident fields at a planar dielectric interface of two media are

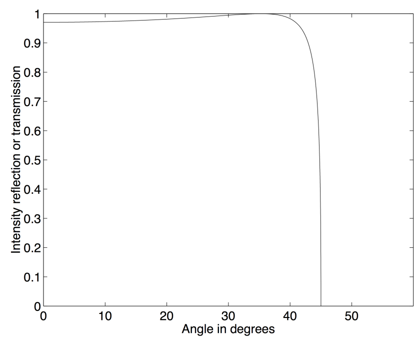
$$r_s(\theta) = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \quad r_p(\theta) = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

where $n = n_2/n_1$ is the relative refractive-index, and θ measures the angle of incidence.

(a - 2pt) Explain what it is and *derive* an equation for the critical angle θ_{cr} for total internal reflection.

(b - 2pts) Starting from the equation for $r_s(\theta)$ prove that the magnitude (or absolute value) of the reflectivity is unity for $\theta > \theta_{cr}$.

The following figure shows either the reflectance or transmittance for either internal of external reflection at a dielectric interface as a function of incident angle θ, and for either an TE-polarized or TM-polarized incident field.



(c - 2pts) By inspecting the curve explain which combination of the above options the plotted curve corresponds to.

(d - 1pts) Estimate the relative refractive index.

(e - 1pts) Consider reflection at normal incidence from the interface corresponding to the above figure. Explain if the reflected wave acquires phase change π upon reflection.

(f - 2pts) Let the material boundary be located in plane $z = 0$, and let the incident plane wave propagate along vector $\hat{s} = -(\hat{k} + \hat{\imath})/\sqrt{2}$. Specify plane of incidence and give the direction of oscillation for the s-polarized wave.

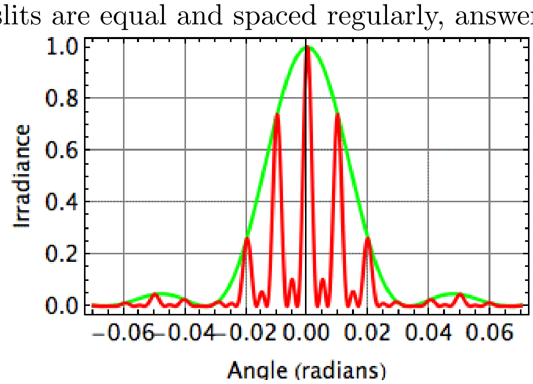
4. (10pts) This question deals with Fraunhofer and Fresnel diffraction.

(a - 2pts) Describe the difference between the Fresnel and Fraunhofer diffraction regimes.

(b - 2pts) Helium-neon laser light ($\lambda = 632.8$nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?

(c - 2pts) A beam of monochromatic green light is diffracted by a slit of width 0.5 mm. The diffraction pattern forms on a wall 2.0 m beyond the slit. The distance between the positions of zero intensity on opposite sides of the central bright fringe is 4 mm. Calculate the wavelength of the light.

(d - 4pts) The following figure shows Fraunhofer diffraction pattern from an N-slit aperture. Assuming that all slits are equal and spaced regularly, answer the following questions:



1) Which of the curves shown corresponds to a single-slit?
2) How wide is a single slit? Express you answer in units of λ.
3) How many individual slits constitute the whole aperture?
4) How far apart are the slits (in units of λ)?

5. (10pts) This question deals with polarization, Jones calculus and Faraday magneto-optic effect.

(a - 2pts) Faraday rotator can be described by the following Jones matrix:

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Describe the effect of this element on the light that is linearly polarized. How is the parameter α related to the magnetic field in the rotator?

(b - 2pts) Write down a Jones vector for light linearly polarized, oscillating along a direction making 30 degrees with the horizontal axis. Assume light with this polarization goes through the rotator described above. Calculate the output polarization state for $\alpha = \pi/4$.

(c - 2pts) Describe (in words) how two linear polarizes together with a Faraday rotator can be arranged into an optical isolator. Explain how it works.

(b - 4pts) Describe how a slab of a uniaxial-crystal material can be used to create a quarter-wave plate.
1. What should be the difference between the transmitted phases of ordinary and extraordinary rays?
2. Given the refractive indices n_e and n_o, what should be the thickness of a (zero-order) wave plate?
3. How should the crystal optic axis be oriented with respect to the direction of light propagation?
4. What is the oscillation direction, with respect to optic axis, of the extraordinary ray?