Name:	

OPTI 310, Fall 2015 **Final Exam** Prof. M. Kolesik

Dec. 16, 2015.
10:30 am - 12:30 pm

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer ALL questions. Show supporting arguments — unjustified answers receive reduced credit!

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8$ m/s $\quad\quad \hbar = 1.05 \times 10^{-34}$ Js

$\frac{\pi w_0^2}{\lambda} \quad\quad \frac{\pi w_0^2}{2}$

$\frac{1}{2}\epsilon_0 c n E_0^2$

$\sqrt{1\lambda L} \quad \sqrt{2\lambda L} \quad \sqrt{3\lambda L}\ldots \quad L = \left(\frac{1}{h} + \frac{1}{h'}\right)^{-1}$

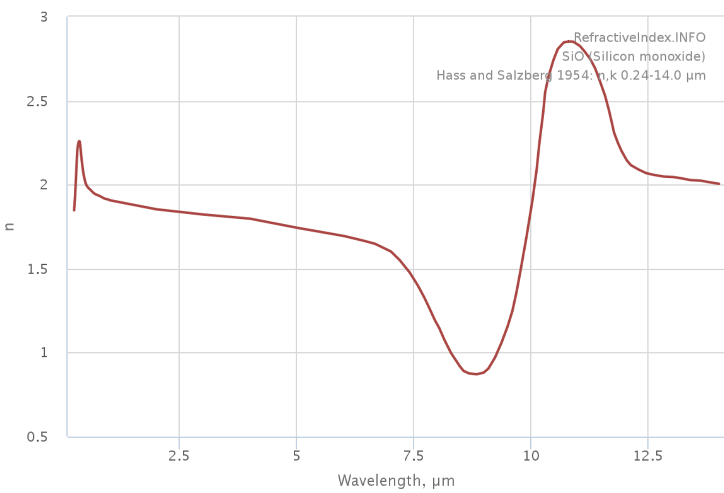
$\nu = \frac{c}{\lambda} = N\nu_{FSR} \quad\quad \mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} \quad\quad \frac{c}{2d} \quad\quad RP = N\mathcal{F}$

Score:	/out of 60 possible points

1. This problem covers a variety of topics from the class. No long calculations are required.

(a - 3pts) An electromagnetic plane-wave propagates in vacuum, and its wave-vector is $\vec{k} = \frac{2\pi}{\sqrt{3}\lambda}(\hat{k} + \hat{j} + \hat{i})$. Assuming that the wave polarization is linear, find at least one possible pair of electric and magnetic vectors $\hat{e}$ and $\hat{b}$, (that give the oscillation direction of their respective fields) with the following properties:
a) $\hat{e}$ has zero z-component
b) $\hat{e}$ and $\hat{b}$ are both unit vectors

(b - 6pts) This figure shows real or imaginary part of index of refraction of silicon monoxide



1) Does the curve show the real or imaginary part of the refractive index?
2) Identify two wavelength regions in which this material exhibits significant absorption.
3) Identify a region in which this material exhibits anomalous dispersion.
4) Identify the transparency window where the absorption is low.
5) Estimate the critical angle for TIR (with air) at $\lambda = 2.5\mu$m.
6) What would be the Brewster angle (from air) for the same wavelength?

(c - 1pts) Write down one-dimensional scalar wave equation for propagation along the coordinate axis x with wave velocity u.

(d - 2pts) The following formulas give an exact and/or approximate solution to the three dimensional scalar wave equation:

$$\text{A:} \quad \frac{1}{\sqrt{\rho}} \cos[k\rho - \omega t] \qquad \text{B:} \quad \frac{1}{r} \exp[i(kr - \omega t)]$$

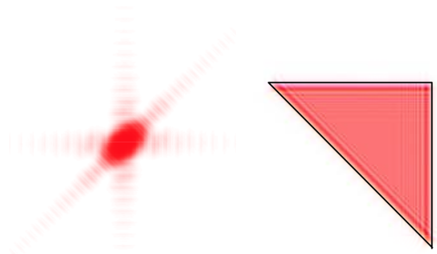
1) Which of these represents a spherical and which a cylindrical wave?
2) Which is exact and which is only approximate (good for large distances from its source)?
3) If the wave velocity is c, give the dispersion relation that must be satisfied between ω and k.

(e - 2pts) Pulsed laser beam has wavelength $\lambda = 800$ nm and a very large peak power of $P = 10^{12}$ W. Calculate the energy per photon of the laser light and the peak photon flux (number of photons per unit of time).

(f - 2pts) Explain the notion of the Fresnel zone, and describe qualitatively the function of the so called Fresnel zone plate which concentrates a collimated beam to an approximate focal point similar as a conventional lens.

(i - 2pts) Describe what a quarter-wave plate does to the polarization state of a linearly polarized beam in two cases:
A) the oscillation direction of the incident light is aligned with the fast or slow axis of the plate
B) the oscillation direction makes 45 degree angle with the axes of the plate

(j - 2pts) These patterns show diffraction of a plane wave on a triangular aperture. Which is Fraunhofer and which is Fresnel diffraction? Which picture was 'taken' at a distance closer to the screen?



2. In this problem, consider electromagnetic field as described by Maxwell equations. Assume the medium is dielectric and non-magnetic, and allow for presence of free charges and currents.
(a - 1pts) Write down the Faraday law in the differential form.

(b - 1pts) Write down the Ampere law in the differential form.

(b - 2pts) Write down both magnetic and electric Gauss law in the component form, i.e. in terms of $D_x, D_y, D_z, B_x, B_y, B_z$.

(b - 2pts) Consider the following plane-wave solutions in complex representation for the electric and magnetic fields

$$\vec{E}(\vec{r}, t) = \hat{j} E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \qquad \vec{B}(\vec{r}, t) = \hat{k} B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{1}$$

where ω is the field angular frequency.
1) Determine $\vec{k}$ in terms of the angular frequency so that the above pair can be a plane-wave solution to the Maxwell equations in vacuum.
2) Derive the relation between field amplitudes E_0 and B_0.

(c - 2pts) Specify the real physical fields corresponding to the above complex representation, and calculate the corresponding time-dependent Poynting vector $\vec{S}$. Show the general Poynting vector formula and apply it to this specific case.

(d - 2pts) Calculate the time-averaged irradiance of the above plane wave. Express it in terms of E_0 and universal constants.

3. This problem concerns reflection/refraction of a plane wave at a planar material interface, and properties of optical coatings. The Fresnel formulas for the reflection and transmission coefficients for s-polarized and p-polarized incident fields at a planar dielectric interface of two media can be written as follows (in arbitrary order!):

$$A = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad B = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad C = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \qquad D = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

where n_i, n_t are the relative refractive indices of the "incident" and "transmitted" medium, and θ_i, θ_t stand for the angle of incidence and transmitted angle, respectively. The convention here is the same as in Hecht.

(a - 2pts) Identify which quantities A, B, C, D represent the amplitude reflection and transmission coefficients r_s, r_p, t_s, t_p.

(b - 2pts) Consider $n_i > n_t$. Demonstrate that at the critical angle of incidence, the transmission coefficients attain "universal values" of 2 and $2n_i/n_t$, while the reflection coefficients become unity in absolute values.

(c - 2pts) The material interface coincides with the coordinate plane $y = 0$. The propagation direction of the incident wave is $\hat{u} = (0, 1, -1)/\sqrt{2}$.
1) Specify the plane of incidence.
2) Specify a possible electric polarization vector $\hat{e}_s$ for the s-polarized wave
3) Specify a possible electric polarization vector $\hat{e}_p$ for the p-polarized wave

(d - 2pts) What is a Brewster window? Describe its function and/or possible application.

(e - 1pts) A substrate glass with $n = 1.7$ is to be coated to minimize reflection of light with $\lambda = 500$nm at normal incidence. How would you choose the parameters of an ideal AR coating?

(f - 1pts) Explain the principle of high-reflectivity coatings (or Bragg mirrors).

4. The transmittance for an ideal (i.e. assuming no losses) Fabry-Perot illuminated at normal incidence is given by the Airy function

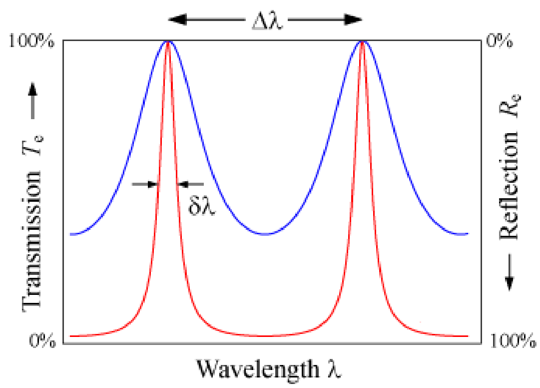
$$\mathcal{T} = \left(\frac{I_T}{I_0}\right) = \frac{1}{1 + F \sin^2(\Delta/2)}, \tag{2}$$

where $F = 4R/(1-R)^2$ is the coefficient of finesse, and $\Delta = 2kd$ is the phase factor, d being the spacing between the mirrors, the mirror reflectance is R, and the incident wavelength $\lambda = 1.0$ microns. The order of interference is $N = 2000$ when the resonance is achieved for the distance d_0 between the mirrors.

(a - 1pt) What is the mirror distance d_0?

(b - 2pts) Make a qualitative sketch of the transmittance as a function of the mirror distance d when d varies around d_0 such that the order of interference changes by ± 1. Indicate important features and values.

(c - 3pts) The following figure shows the transmission and reflection for two different F-Ps.



1) One parameter is the same in bot FPs. Which is it?
2) Which curve represents FP with a higher R?
3) Estimate R for the blue curve.
Which feature of the plot are you going to use?
4) Estimate the finesse $\mathcal{F}$ for the FP shown in red.
5) What can you say about the losses in these FPs?

(c - 2pts) Explain the notions of division of amplitude and division of wavefront. Give examples.

(d - 2pts) Make a sketch of both a Michelson and Mach-Zehnder interferometers, indicating the light paths, and differentiating between the splitters and mirrors.

5. This question deals with Fraunhofer and Fresnel diffraction.

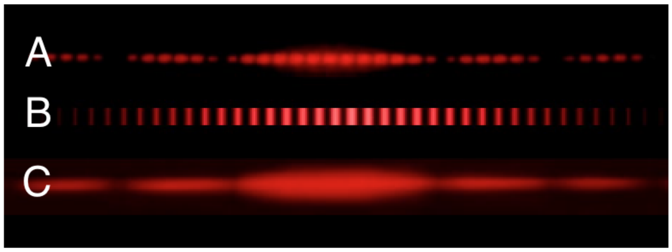
(a - 2pts) Describe the effect known as Poisson bright spot (or spot of Arago). Give a qualitative explanation with the help of the notion of Fresnel zone.

(b - 2pts) A circular aperture of unknown radius a is illuminated by a point source of $\lambda = 1$ micron from a distance of $h' = 1$m on the axis. What is the radius of the aperture if the intensity of the diffracted light vanishes at the observation point located at the distance $h = 1m$ from the screen. Explain how many Fresnel zones need to be open.

(c - 2pts) The Fraunhofer patterns for diffraction on the single and multiple slits are described by the formula:

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 \quad , \quad \beta = \frac{\pi w \sin\Theta}{\lambda} \quad , \quad \gamma = \frac{\pi d \sin\Theta}{\lambda}$$

The following figure shows Fraunhofer diffraction patterns from single- and double-slit apertures.



1) Which of A,B,C, corresponds to a single-slit?
2) Which has the narrowest slit(s)?
3) Describe what would happen to this picture if the incident light was green instead of red.

(d - 2pts) Describe Huygens-Fresnel principle (HFP).

(e - 2pts) Express HFP mathematically for a one-dimensional case of diffraction of a plane wave falling at an angle α onto a simple slit $2w$ wide: Show an integral expression for the amplitude in the far field.