Practice Final Answers

Note: numerical results are rather approximate in this document. Also, watch for mistakes :-)

1.

a) note: solution is not unique

$$\hat{e}_1 = (0,1,0) \qquad \hat{e}_2 = \frac{1}{\sqrt{2}}(1,0,-1)$$

b)

1. Red is the real part.
2. wavelength longer than about 150nm
3. In the vicinity of $\approx$ 90nm
4. $\sin\theta_c = 1/1.5 \qquad \theta_c \sim 42$deg.
5. $\tan\theta_B = 1.5 \qquad \theta_b \sim 56$deg

c) Features to show: vector, three-dimensional Laplacian, wave speed equal to that of light:

$$(\partial_{xx} + \partial_{vv} + \partial_{zz} - \frac{1}{c^2}\partial_{tt})\vec{E}(x,y,z,t) = 0$$

d) Features to show: NO dot product, inverse dependence on radial distance:

$$\frac{1}{r}\exp[ikr - i\omega t]$$

e) Useful to remember: 1μm wavelength corresponds to about 1.24eV which in turn is about 1.6×10^{-19}J

$$E_{ph} = \hbar\omega \sim 3 \times 10^{-19} J$$

f) Rayleigh range can be characterized in different ways: distance from the beam waist to where the intensity decreases to one half, or distance where "radius" increases by a factor of root of two.

$$z_r = \frac{\pi w_0^2}{\lambda} \sim 5m$$

g) This relation between power and peak intensity of a Gaussian beam is essentially the definition of the effective cross-section area:

$$P = AI \qquad \text{where} \qquad A = \frac{\pi w_0^2}{2}$$

h) The formula needed here is

$$I = \frac{1}{2}\epsilon_0 c n E_0^2$$

Using the previous result

$$E_0 = \sqrt{\frac{2P}{A\epsilon_0 cn}}$$

i) Various answer acceptable here. HW plate "flips" the polarization of the incident linearly polarized light w.r.t. the plate axis, thus "rotating" the electric oscillation direction by twice the angle between it and the plate axis.

j) The features to indicate: two "streaks" of diffracted light, in perpendicular direction, with the central spot being a rectangle with the aspect ratio inverse of that of the slit.

2.

a)

$$\nabla \times \vec{H} = +\partial_t \vec{D} \qquad \vec{B} = \mu_0 \vec{H} \qquad \nabla.\vec{B} = 0$$
$$\nabla \times \vec{E} = -\partial_t \vec{B} \qquad \vec{D} = \epsilon_0 \epsilon_r \vec{E} \qquad \nabla.\vec{D} = 0 \tag{1}$$

b) From Faraday:

$$i\vec{k} \times \hat{e} E_0 = (-1)(-i\omega)\hat{b} B_0 \qquad \hat{b} = \hat{k} \times \hat{e} \qquad B_0 = E_0/c$$

c) From Gauss:

$$0 = \nabla.\vec{E} = i\vec{k}.\hat{e}E_0 \exp[\ldots]$$

for this to be zero, $\vec{k}.\hat{e}$ must vanish, meaning that the two are perpendicular.

d) The above: $\hat{k}, \hat{e}, \hat{b}$. Or even more lazy: $\hat{\imath}, \hat{\jmath}, \hat{k}$.

3.

a) Plane of incidence is given by two vectors: Incident wavevector and material interface normal. Alternatively, incident and reflected wavevectors. Transmitted wavevector can also be used. For normal incidence the choice of the plane of incidence is not unique.

b) S-polarized wave oscillates (talking about the electric field, always) in direction perpendicular to the plane of incidence. The P-polarized wave oscillates in the plane of incidence. Other names: S = TE, P = TM, i.e. transverse electric and transverse magnetic.

c) The first is TE. Look for the homogeneous combination of i and t in angle and index. If you do not remember, make an argument that the second *can* attain zero (Brewster) so it must be p-polarized.

d) One way to show this:

$$r_p = 0 \to n_t \cos\theta_i = n_i \cos\theta_t$$

... and using formula for Brewster,

$$\sin\theta_i = \tan\theta_i \cos\theta_i = \frac{n_t}{n_i}\cos\theta_i = \cos\theta_t$$

which means that the two angles make up together 90 degrees.

e) At critical incidence, the transmitted wave propagates along the interface, so $\theta_t = \pi/2$. Insert into Snell's, to get $n_i \sin\theta_i = n_t.1$

f) Features to show: External therefore no critical, therefore extent of the graphs from zero to 90. Small negative value at zero, zero crossing for TM (P), and plus minus one at 90. Note that the sign (and plot) of P depends on the convention (Hecht vs Fowles). Use either one.

4.

a) Full transmission (because of zero losses).

b) For this one, it is probably easiest to convert distances to $\Delta = 2kd$. The middle will correspond to $\Delta = 4000\pi$, and the two extremities to $\Delta = 999 \times 4\pi$ and $\Delta = 1001 \times 4\pi$. The plot will thus extend for 4π on each side, and show two peaks on each side of the central one because the plot in terms of Δ is periodic with 2π.

Other feature: Max reaching unity, min small, 1/81, because $F = 80$. Sharp peaks, flat valleys.

c) The ratio of: the distance between the peaks in the above picture over the width (FWHM) of the peaks. This quantity characterizes the resolution capability of the FP.

d) The peaks would be lower and fatter.

e) Peaks will be skinnier, valleys deeper and flatter.

5.

a) Area in the aperture over which the phase of the secondary HF source does not vary more than by π.

b) Use $R_n = \sqrt{nL\lambda}$. Four zones open.

c) Nearly zero, because of the even number of open zones.

d) Green envelope originates in the single slit diffraction. The distance between the skinny peaks originates in the single slit width. The width of the narrow peaks correspond to the number of slits. To correlate to the given formula: w is the slit width, d id the slit center to center distance.

e) Three. Look for how many Peak-to-peak spaces fit within the side of the green envelope central lobe: There would be a red peak at 0.1 radians. This would be the third one. This means that γ changes three times faster than β, so the ratio $d/w = 3$.

f) Four. Count red peaks from the central to the next. If you forget exactly how: Recall that for the two-slit pattern there is only the green envelope plus a simple periodic pattern (no substructure between red peaks). So you count from the central to the next big one.

g) Ten wavelengths. From the zero at 0.1 radians:

$$\frac{\pi w}{\lambda} \sin(0.1) = \pi \qquad w = 10\lambda$$