Example: Standing Wave, Energy Density, Poynting, ...

Given: Two counter-propagating PWs:

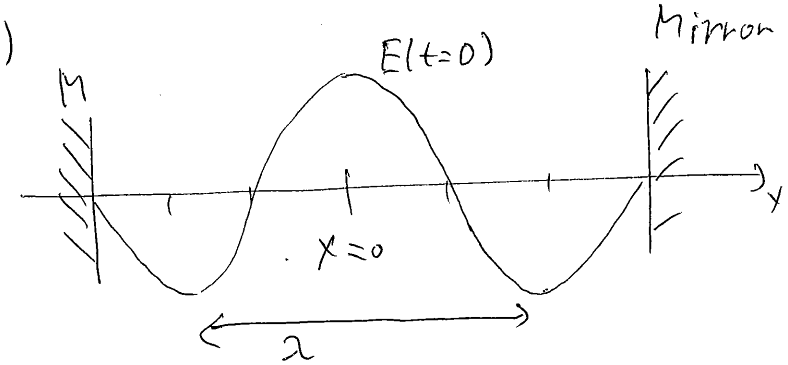
$$\vec{E} = \hat{j} E_0 \left(\cos(+kx - \omega t) + \cos(-kx - \omega t) \right) \qquad k = \frac{\omega}{c}$$

A)

$\vec{B} = ?$

using $\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$ get:

$$\vec{E} = \hat{j}\, 2E_0 \cos(\omega t)\cos(kx)$$



use Faraday:

$-\partial_t \vec{B} = (\nabla \times \vec{E})$ only non-zero is: $(\nabla \times \vec{E})_z = \partial_x E_y - \cancel{\partial_y E_x}^{\,0}$

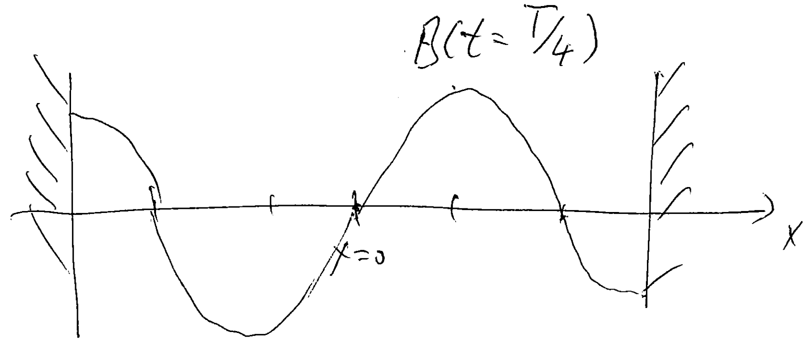
$$(\nabla \times \vec{E})_z = 2E_0 \cos(\omega t)(-1)\sin(kx)\frac{\omega}{c}$$

so:

$$\partial_t \vec{B} = \hat{k} \frac{E_0 \omega}{c}\, 2\cos(\omega t)\sin(kx)$$

integrate to get:

$$\vec{B} = \hat{k} \frac{2E_0}{c} \sin(\omega t)\sin(kx)$$



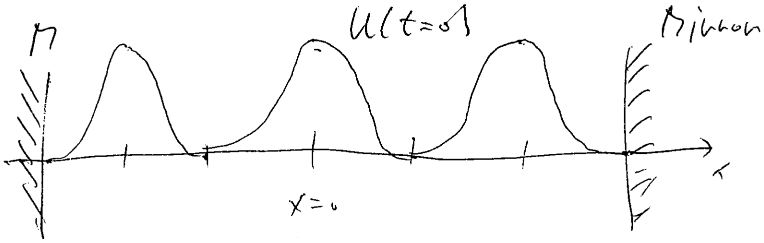
Ex. cont:

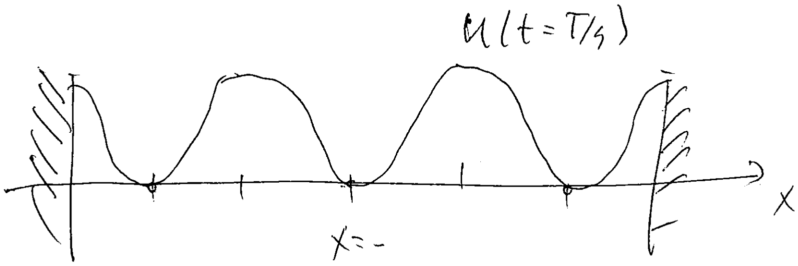
B) What is the energy density?

$$U = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = 2\varepsilon_0 E_0^2 \cos^2(\omega t)\cos^2(kx) + 2\varepsilon_0 E_0^2 \sin^2(\omega t)\sin^2(kx)$$

Time averaged:

$$\langle U \rangle_T = 2\varepsilon_0 E_0^2 \frac{1}{2}\cos^2(kx) + 2\varepsilon_0 E_0^2 \frac{1}{2}\sin^2(kx) = \varepsilon_0 E_0^2$$

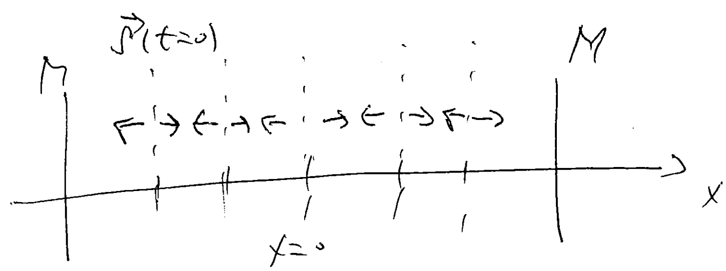




C) Calculate Poynting Vector, and its time-average

$$\vec{S} = ? \qquad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\hat{j}\times\hat{k}\,\frac{E_0^2}{c}\underbrace{2\sin(kx)\cos(kx)}_{2\sin(2kx)}\underbrace{2\sin(\omega t)\cos(\omega t)}_{2\sin(2\omega t)}$$

$$\vec{S} = \hat{\imath}\,\varepsilon_0 c E_0^2 \sin(2kx)\sin(2\omega t)$$



$$\langle \vec{S} \rangle_T = 0$$