The following material reviews material boundary conditions for electric and magnetic fields. We will need them to derive our main objective of this section which is **Fresnel equations**.

Boundary conditions for EM fields:

Maxwell equations impose this:

Sharp material interface:

- Induction fields **D**, **B**: Their *normal* components are continuous.
- Intensity fields **E**, **H**: Their *tangential* components are continuous.
- continuity of $D_\perp$, implies discontinuity in $E_\perp$

Application:

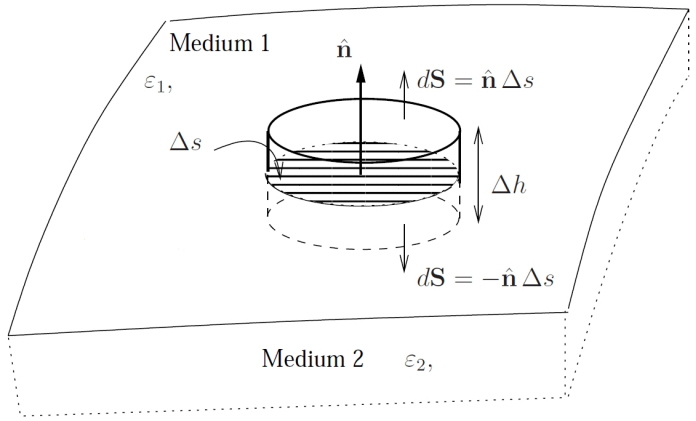
- Calculation of reflection from and transmission through a material interface
- Calculation of EM modes in waveguides
- ... and many other

Boundary conditions for electric (and magnetic) induction:

Important: We assume that there are no free charges, and no currents (volume or surface densities).

All follows from the Gauss law:

$$\oint_S \mathbf{D}.d\vec{S} = \int_V \rho dV = 0 \qquad \oint_S \mathbf{B}.d\vec{S} = 0$$



$$0 = \oint_S \mathbf{D}.d\vec{S} = \mathbf{D}_1.\hat{n} + \mathbf{D}_2.(-\hat{n}) = \Delta s(D_{n1} - D_{n2})$$

$$D_{n1} - D_{n2} = 0$$

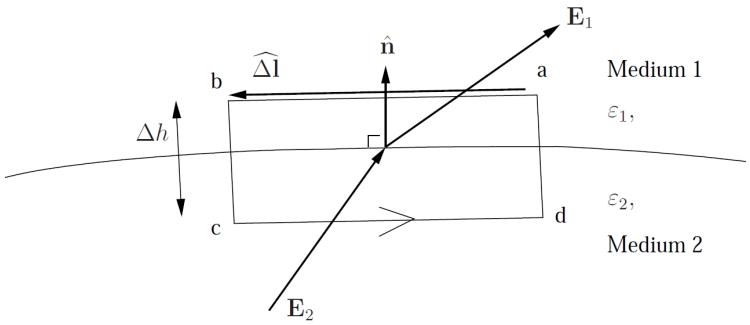
The same argument leads to:

$$B_{n1} - B_{n2} = 0$$

Boundary conditions for electric (and magnetic) intensity:

This time we use Faraday (Ampere):

$$\oint_C \mathbf{E}.d\vec{l} = -\oint_S \partial_t \mathbf{B}.d\vec{S}$$



$$\oint_C \mathbf{E}.d\vec{l} = \int_a^b \mathbf{E}_1.d\vec{l} + \int_c^d \mathbf{E}_2.d\vec{l} = \mathbf{E}_1.\Delta\vec{l} + \mathbf{E}_2.(-\Delta\vec{l}) = 0$$

and because only tangential components contribute, we have:

$$E_{t1} - E_{t2} = 0$$

Similar argument based on Ampere leads to

$$H_{t1} - H_{t2} = 0$$

Interface conditions summary:

A) Component form (the form we use in OPTI-310, remember this) :

$$\begin{aligned} D_{n1} - D_{n2} &= 0 \\ B_{n1} - B_{n2} &= 0 \\ E_{t1} - E_{t2} &= 0 \\ H_{t1} - H_{t2} &= 0 \end{aligned}$$

B) In words:

1. Normal components of B and D are continuous at the material interface
2. Tangential components of E and H are continuous at the material interface

Note: Normal and tangetial is meant with respect to the normal vector of the interface