Name:	

OPTI 310, Fall 2018 **Final Exam** Prof. M. Kolesik

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer ALL questions. Show supporting arguments — unjustified answers receive reduced credit!

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$$\hbar\omega = h\nu \quad , \quad \hbar = 1.054 \times 10^{-34}\text{Js} \quad , \quad \hbar\vec{k}$$

polarizers:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$

plates:

$$\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \quad , \quad \begin{pmatrix} 1 & 0 \\ 0 & +i \end{pmatrix} \quad , \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \pm i \\ \pm i & 1 \end{pmatrix} \quad , \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_n = \sqrt{\lambda n L}$$

$$|U_p| = |U_1| - |U_2| + |U_3| - |U_4| + |U_5| + \dots$$

$$|U_p| = |U_1|/2 + (|U_1|/2 - |U_2| + |U_3|/2) + (|U_3|/2 - |U_4| + |U_5|/2) + \dots \approx |U_1|/2$$

$$A + T + R = 1 \quad , \quad I_{max}/I_0 = \frac{T^2}{(1-R)^2} = \left(\frac{1-A-R}{1-R}\right)^2 \quad , \quad \frac{c}{2nd} \quad , \quad \frac{4R}{(1-R)^2} \quad , \quad \frac{\pi}{2}\sqrt{F}$$

Score:	
	/out of 60 possible points

P1: (10pts) This qualitative problem concerns the physics behind the phenomenon of diffraction

A) (2pts) Explain the idea of Huygens-Fresnel principle.

1. Addition of waves from:
2. Secondary sources

B) (2pts) Write a general diffraction integral representation of the Huygens-Fresnel principle, and identify the physical meaning of its various parts.

$$\int_{\text{APERTURE}(y)} \frac{1}{r}\exp[ikr]\,dy$$ r = distance (y, observation point)

C) (1pt) Explain the notion of a Fresnel zone for diffraction on an aperture.

Area of the same (approx.) phase

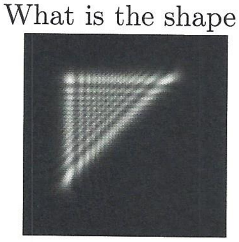
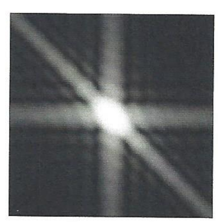
D) (1pt) What is the difference between Fraunhofer and Fresnel diffraction regimes?

Far field NOT Far

E) (1pt) What happens to the far-field diffraction intensity pattern if we increase the size of the diffraction aperture by a factor of 2 (two)?

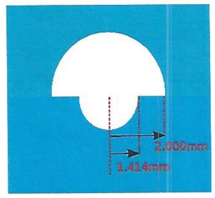
Gets NARROWER in angular extent

F) (2pt) Identify the diffraction patterns in these pictures: which is in the Fresnel and which is in the Fraunhofer regime? What is the shape of the aperture?

Fresnel   Shape  Fraunhofer

G) (1pt) What happens to the pattern on the left if we use a laser with a smaller wavelength? Are the fringes to be coarser or finer?

P2: (10pts) Plane wave with wavelength of $\lambda = 500$ nm and irradiance of $I_0 = 33$ W/m^2 illuminates an opaque screen with an aperture shown in the figure. The observation point is on axis, 4 meters from the screen.



A) (1pt) An approximate formula for the observed field amplitude U_p in terms of amplitudes U_1, U_2, U_3 ... from the Fresnel zones 1,2,3,..., with illuminated fractions f_1, f_2, f_3 ... reads

$$U_p \sim f_1 U_1 - f_2 U_2 + f_3 U_3 - f_4 U_4 \ldots$$

Explain the origin of the alternating signs.

Fresnel zones have "opposite" phases

B) (1pt) State the physical reason why we can say that if Fresnel zones are fully illuminated, then their corresponding amplitudes are approximately equal: $|U_1| \approx |U_2| \approx |U_3| \approx \ldots$

FZs have the same area

C) (1pt) State the relation between the field amplitude U_1 caused by illumination through a single zone, and the field amplitude U_0 due to un-obstructed illumination by a plane wave.

$U_0 \doteq \frac{1}{2} U_1$

D) (4pts) For the screen shown in the figure, calculate the radii R_i $i = 1, 2, \ldots$ of the first few Fresnel zones, and determine which are illuminated, or what fraction f_i of each zone is illuminated.

$R_1 = \sqrt{1 \lambda L} = \sqrt{2}$ mm $\qquad f_1 = 1$

$R_2 = \sqrt{2} R_1 = 2$ mm $\qquad f_2 = \frac{1}{2}$

E) (3pt) Apply the above results and calculate the irradiance through the aperture shown at an on-axis point 4 meters away from the screen.

$U = 1 \cdot U_1 - \frac{1}{2} U_2 = 1 \cdot U_1 - \frac{1}{2} U_1 = \frac{1}{2} U_1 = U_0$ (from C)

$I = I_0$

P3: (10pts) Consider a system consisting of:
1. Ideal polarizer with the transmission axis making angle of 45 degrees with the vertical.
2. Half-wave plate designed for the the red light ($\lambda_r = 780$ nm). Its fast axis is vertical.
3. Ideal polarizer with the transmission axis perpendicular to that of the first polarizer.

A) (3pts) Write the Jones matrix for each of the elements.

$$P_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad HW = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad P_2 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

B) (3pts) Calculate the Jones matrix for the whole system.

$$M = P_2 \, HW \, P_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

C) (3pts) Red light ($\lambda_r = 780$ nm, intensity I_0) polarized linearly along 45deg illuminates the system. What is the intensity and the polarization state of the light transmitted through the system? You can either use Jones calculus to obtain the answer, or consider the effect of each element in order.

$$JV: \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{2}\begin{pmatrix} 2 \\ -2 \end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = LP_{-45°}$$

D) (1pts) Assume that the half-wave plate is made of a hypothetical birefringent material in which neither ordinary nor extraordinary index of refraction depends on the wavelength. This time illuminate the system with violet light ($\lambda_v = 390$ nm). What happens? Hint: The key is to figure out what effect has the plate on the violet light.

$$\lambda_v = \frac{1}{2}\lambda_r \Rightarrow HW \rightarrow \text{Full-wave plate} = \text{identity matrix}$$

$$\widetilde{HW} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\tilde{M} = P_2 \, \widetilde{HW} \, P_1 = P_2 P_1 = \emptyset$$

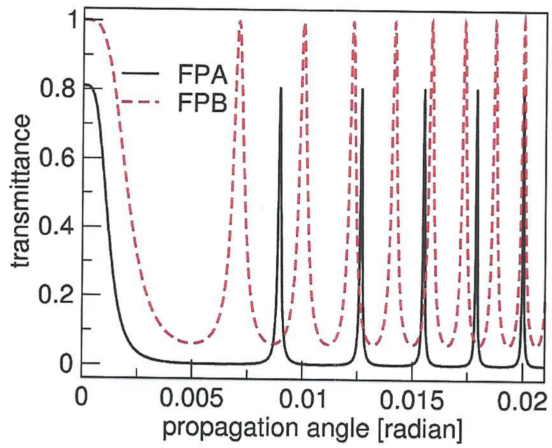
NO LIGHT

P4: (10pts) This problem deals with the Fabry-Perot (FP) interferometer. We have derived the following formula for its transmittance

$$I/I_0 = \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2(\Delta/2)}$$

where $\Delta = 2kd\cos\theta$ (the medium between mirrors is air, and we neglect additional phase change δ_r from the reflection on the mirrors).

A) (4pts) The following figure shows the transmittance of two different FPs, labeled FPA and FPB, as functions of the propagation angle (in radians), when illuminated by light with wavelength of $\lambda = 400$ nm.



1. One of the FPs has no losses, the other has mirrors with non-negligible absorption A. Which is which?

B

2. Which interferometer has a larger mirror distance?

A

B) (6pts) Assume that the mirror distance in a Fabry-Perot is 0.5 mm, reflectance of each mirror is equal to 0.9, and the transmittance is 0.05. Evaluate and explain the physical meaning of

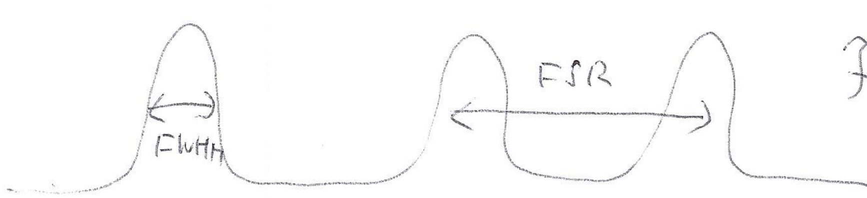
1. Free spectral range

$FSR = \frac{c}{2d}$

2. Finesse

$\mathcal{F} = \frac{\pi}{2}\sqrt{F}$; $F = \frac{4R}{(1-R)^2}$

3. Make a qualitative sketch of transmitted intensity versus light frequency, and use it to mark the free spectral range together with the quantity that determines the finesse.



$\mathcal{F} = \frac{FSR}{FWHM}$

P5: Maxwell equations, plane waves, and light polarization.

A) (2pt) Finish these equations (for free space):

$$\partial_t B_y = \ldots -(\nabla \times E)_y \qquad \partial_z B_x - \partial_x B_z = \ldots (\nabla \times B)_y = \frac{1}{c^2}\partial_t E_y$$

$$-\partial_t B_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

B) (2pt) What is the polarization state of this plane wave?

$$\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t) + 2\hat{\jmath} E_0 \cos(kz - \omega t + \pi/4)$$

$\varphi_y = \pi/4 \quad \varphi_x = 0 \quad \varphi_{yx} = \pi/4 \Rightarrow$ "L"

$A_x \neq A_y$

$\Rightarrow$ LE

C) (2pts) Calculate the magnetic field for the electric field given in the previous problem.

$$\vec{B} = (\hat{k} \times \hat{\imath}) \frac{E_0}{c} \cos(kz - \omega t) + (\hat{k} \times \hat{\jmath}) \frac{2E_0}{c} \cos(kz - \omega t + \pi/4)$$

D) (2pts) Give the complex representation of the wave from problem B).

$$\vec{E} = \hat{\imath} E_0 \exp[ikz - i\omega t] + 2\hat{\jmath} E_0 \exp[ikz - i\omega t + i\pi/4]$$

$$\vec{E} = (\hat{\imath} E_0 + 2\hat{\jmath} E_0 e^{i\pi/4}) \exp[ikz - i\omega t]$$

E) (2pts) Specify the Jones vector of the wave you gave as answer in the previous problem.

$$JV = \begin{pmatrix} 1 \\ 2e^{i\pi/4} \end{pmatrix} = \begin{pmatrix} 1 \\ 2\frac{1+i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{2} + i\sqrt{2} \end{pmatrix} \Rightarrow \text{NOT C}$$

"L"

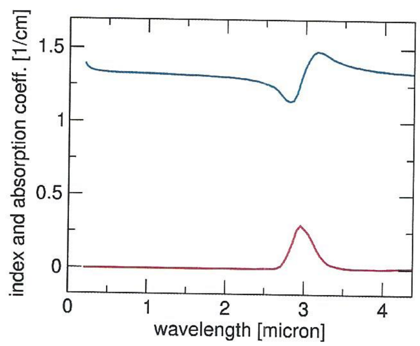
P6: This problem samples a variety of topics.

A) (2pt) Design a single-layer anti-reflection coating for a plate with refractive index of $n_s = 2.409$ in air for the normal incidence of light with $\lambda = 589$ nm. What should be the index of refraction n_c of the coating layer? What should be the thickness h of the coating?

1) $n_c = \sqrt{1 \cdot n_s} = \sqrt{2.409}$

2) $h = \frac{\lambda}{4 n_c}$

B) This figure shows refractive index $n(\lambda)$ and absorption coefficient $\kappa(\lambda)$ of water as functions of wavelength.



B1) (1pt) Which curve represents index of refraction and which the absorption coefficient?

B R

B2) (1pt) Identify in this figure the region of anomalous dispersion.

$\lambda \doteq 3\,\mu m$

B3) (1pt) If these properties, $n(\lambda)$ and $\kappa(\lambda)$, should be approximated in the framework of the Lorentz-oscillator medium model, at least how many oscillators do we need?

one, two = better

B4) (1pt) What would be (approximately) the resonance angular frequency of the oscillator with the biggest oscillator strength?

$\lambda \approx 3\,\mu m$

C) (2pts) Consider total internal reflection. Describe in words, qualitatively, the electromagnetic field in the low-index medium in close vicinity of the interface. What can you say about the direction of the Poynting vector?

WAVE = EVANESCENT = exponentially decreasing

$\vec{S}$ = along the interface

D) (2pts) Laser beam with power $P = 20W$ reflects (at normal incidence) from a perfect mirror. Give the force exerted on the mirror. How many photons are reflected per second?

$F = \frac{2P}{c}$

$\phi \hbar\omega = P \Rightarrow \phi = \frac{P}{\hbar\omega}$