Name:	
Score:	/out of 40 possible points

OPTI 310, Fall 2014 **PRACTICE Mid-Term Exam 2** Prof. M. Kolesik

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer all questions.

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$$r_\perp = \frac{n_i \cos\Theta_i - n_t \cos\Theta_t}{n_i \cos\Theta_i + n_t \cos\Theta_t} \quad , \quad t_\perp = \frac{2 n_i \cos\Theta_i}{n_i \cos\Theta_i + n_t \cos\Theta_t}$$

$$r_\parallel = -\frac{n_t \cos\Theta_i - n_i \cos\Theta_t}{n_t \cos\Theta_i + n_i \cos\Theta_t} \quad , \quad t_\parallel = \frac{2 n_i \cos\Theta_i}{n_t \cos\Theta_i + n_i \cos\Theta_t}$$

$$\sqrt{1+a} \approx 1 + \frac{1}{2} a + \dots$$

$$\frac{1}{v_?} = \frac{k(\omega)}{\omega} \qquad \frac{1}{v_?} = \frac{\partial k(\omega)}{\partial \omega}$$

1. An electromagnetic plane wave propagates along the direction given by a vector $(-1, +1, -2)$. It is linearly polarized, with the direction of oscillation of the electric field given by a vector $\hat{e} = (0, 2, 1)/\sqrt{5}$. This wave is incident on a material-air interface from a medium with a refractive index of n. The medium-air boundary coincides with the plane specified by the equation $x - y + z = 1$.

A (1pts) Calculate the unit vector $\hat{k}$ in the direction of propagation. Verify that it is perpendicular to the polarization direction vector.

$$\hat{k} = \frac{1}{\sqrt{6}}(-1, +1, -2) \qquad \hat{e} = \frac{1}{\sqrt{5}}(0, 2, 1) \qquad \hat{e}.\hat{k} = \frac{1}{\sqrt{30}}(0 + 2 - 2) = 0$$

B (1pts) Calculate the unit normal $\hat{n}$ of the interface. Orient this vector such that it points away from the medium from which the wave propagates.

$$\hat{n} = -\frac{1}{\sqrt{3}}(1, -1, 1)$$

C (1pts) Determine the plane of incidence by calculating its normal unit vector $\hat{s}$

$$\hat{s} = \pm\frac{1}{\sqrt{2}}(1, 1, 0)$$

D (1pts) Specify $\hat{e}_s$, the unit polarization vector for the s-polarized wave?

$$\hat{e}_s = \hat{s}$$

E (1pts) Calculate $\hat{e}_p$, the unit polarization vector for the p-polarized wave.

$$\hat{e}_p = \pm\frac{1}{\sqrt{3}}(1, -1, -1)$$

H (1pts) Determine the fractions of power in the incident beam that propagates in the s and p polarized waves.

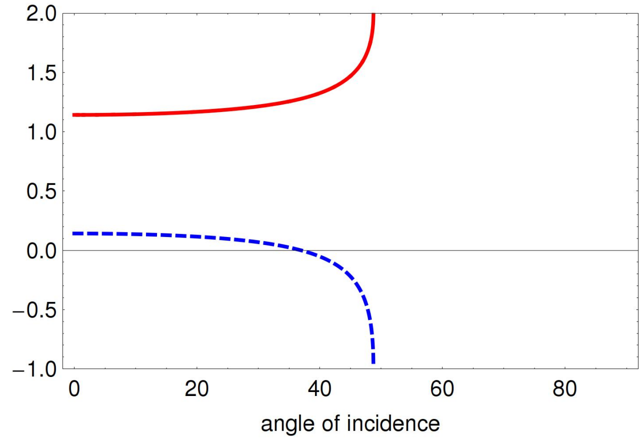
$$P_s = \frac{2}{5} \qquad P_P = \frac{3}{5} \qquad P_s + P_p = 1$$

F (4pts) Determine the transmission coefficients for both polarizations as functions of n. Decide what must be the index of refraction n for the reflected light to be fully polarized. What is then the polarization direction of the reflected light?

$$\cos\theta_i = \frac{2\sqrt{2}}{3} \qquad \sin\theta_i = \frac{1}{3} \qquad \tan\theta_i = \frac{1}{2\sqrt{2}} \qquad n = 2\sqrt{2}$$

Because it is Brewster, the direction of polarization must be $\hat{e}_s$.

2. This problem deals with Fresnel formulas.



A (1 pts) Is this picture showing internal or external incidence? Why?
Internal, because the plot has a sharp feature way below 90 degs, so it must be at the critical angle.

B (4 pts) Which of the quantities r_s, r_p, t_s, t_p quantities are shown? Specify and justify your answer for each curve.

$$t_s \qquad r_p$$

Cant be t_p because is reaches exactly values of 2. Must be r_p because of the zero (at Brewster).

C (2 pts) Show the critical angle in the picture, and estimate it value. Do the same for the Brewster angle.

$$\theta_c \approx 48\text{deg} \qquad \theta_B \approx 37\text{deg}$$

D (3 pts) Describe at least two different ways to estimate the relative index of refraction for the interface in question. Evaluate your estimates.

$$\text{from critical:} \quad \sin\theta_c = 1/n \qquad \text{or from Brewster} \quad \tan\theta_B = 1/n$$

... or from the normal reflection coeff., probably inaccurate.

4. The Lorentz electron oscillator model for light-matter interactions leads to the following expression for the refractive-index $n(\omega)$ experienced by a monochromatic field of angular temporal frequency ω propagating in a medium:

$$n^2(\omega) = 1 + \frac{Nq^2}{\epsilon_0 m_e} \frac{1}{[(\omega_0^2 - \omega^2) - i\gamma\omega]} \tag{1}$$

A (3pts) What is the physical meaning of the six quantities that appear in this formula? Specify the physical dimension of each quantity.

number density, charge, permittivity, mass, resonant frequency, damping
should also specify units:

$$m^{-3} \quad As \quad F/m = As/(Vm) \quad s^{-1} \quad s^{-1}$$

B (3pts) Consider a dielectric medium for which the resonance (angular) frequency lies in the ultraviolet region of the spectrum. Sketch the qualitative variation of real part of $n(\omega)$ versus ω, for angular frequencies much less than that of resonance. Which part of $n(\omega)$, real or imaginary, dominates in the transparency window and why?

function to sketch behaves as

$$1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$

picture should show how the second positive term increases the total value above one starting from zero and going to higher frequencies

C (1pts) Does your sketch correspond to normal or anomalous dispersion? Why?

normal, because $\partial_\omega n(\omega) > 0$, i.e. index increases with frequency

D (3pts) Assume that optical losses can be neglected, and that the refractive index is close to unity, as for example in air at normal conditions. Under this assumption, write the refractive index in the form:

$$n(\omega) = 1 + \frac{A}{(\omega_0^2 - \omega^2)}$$

Calculate the formulas for both the phase and group velocity. Using these formulas, show that the group velocity is smaller than the phase velocity.

$$v_p = \frac{\omega}{k(\omega)} = \frac{c}{n(\omega)} = \frac{c}{1 + \frac{A}{(\omega_0^2 - \omega^2)}}$$

$$(v_g)^{-1} = \frac{\partial k(\omega)}{\omega} \qquad v_g = \frac{c}{1 + \frac{A}{(\omega_0^2 - \omega^2)} + \frac{2A\omega^2}{(\omega_0^2 - \omega^2)^2}}$$

v_g is smaller due to the positive addition in the denominator.