Name:	
Score:	see next page /out of 40 possible points

OPTI 310, Fall 2018 **Mid-Term Exam 1** Prof. M. Kolesik

In-class exam, Friday Sept 28, 2018.
11:00-11:50 am.

Notes for the exam:

1. **This is a closed-book, closed-notes exam.** Calculators (with no text stored) may be used during the tests and final exam. No other form of electronic device may be used (no computers, laptops, PDA's, etc). Cell phones are absolutely prohibited during tests and the final exam. Food and drink are prohibited in the exams.

2. Answer all questions.

3. **Show your work and answers on the exam paper in the space following each question. Take the space available as a hint on how much you should be writing if you approach the problem correctly.** You may use additional paper if you find it necessary: this will be provided so do not bring your own paper into the exam. If you do use extra pages, staple the extra pages to the back of your exam. Make sure your final answers are clearly indicated.

4. On any sketches, make sure that axes are labeled and that important graphical trends are clear (such as amplitude, sign, or spatial considerations, etc.). If they are not clear enough, you may add a few words explaining what trends should be visible in the sketch.

5. Vector quantities should be distinguished by an overarrow such as $\vec{A}$.

CONSTANTS and FORMULAE of potential use in this exam:

$c = 3 \times 10^8$ m/s. (OK to use this value)
$\epsilon_0 = 8.85 \times 10^{-12}$ F/m or A s/(V m) , $\mu_0 = 4\pi \times 10^{-7}$ H/m or V s/(A m)
$\hbar = 1.05 \times 10^{-34}$ Js

$$\frac{\pi w_0^2}{2} \qquad \frac{\pi w_0^2}{\lambda} \qquad w_0\sqrt{1 + (z/z_R)^2} \qquad z\left(1 + \left(\frac{z}{z_R}\right)^2\right)$$

$$\frac{\epsilon_0\epsilon_r}{2}\vec{E}.\vec{E} + \frac{1}{2\mu_0}\vec{B}.\vec{B} \qquad \nabla\nabla. - \Delta \equiv \nabla\nabla. - \nabla.\nabla = ?$$

Score:	/out of 40 possible points

1. (10pts)

A) (2pt) Write a one-dimensional wave equation for a scalar field, $\psi(x,t)$ propagating in a medium with wave velocity v. Show by explicit calculation that

$$\psi(x,t) = B(x + vt)$$

where B is a differentiable but otherwise arbitrary function, is an exact solution to the wave equation. Specify the direction in which this solution propagates.

B) (4pt) A three-dimensional vector field is specified by the following function:

$$\vec{A} = \frac{(\hat{j} + \hat{k})}{\sqrt{2}} A_0 \exp\left[i \frac{2\pi}{\sqrt{2}}(-y + z) \times 10^6 - i\ 6\pi \times 10^{14} t \right]$$

where y, z are in meters, and t is in seconds. Specify:
1) the propagation vector (wave-vector) $\vec{k}$
2) the wavelength
3) the velocity of propagation

C) (2pt) Calculate the divergence of this vector field. Decide if $\vec{A}$ could represent magnetic or electric field in a light wave.

D) (2pt) Determine *the direction* of $\nabla \times \vec{A}$. It is not necessary to calculate its magnitude. You may want to make use of the operator equivalencies discussed in the class.

2. (10pts) Consider Maxwell equations in a non-magnetic, dielectric medium with permittivity ϵ.

A) (2pt) Write the relation between the displacement field $\vec{D}$ and the electric field $\vec{E}$. What are the SI unit of $\vec{E}$ and $\vec{D}$? (Express units in terms of V and A, no need to convert to base units.)

B) (4pt) Write the Ampere and Faraday laws in the differential form explicitly, writing out all vector components (do not use operator ∇, your answer must be in terms of $E_x, E_y, \ldots B_x, B_y \ldots$).

C) (1pt) Write the electric Gauss law in the explicit component form.

D) (3pt) For the electric field given by the formula (standing wave)

$$\vec{E} = E_0 \hat{\jmath} \cos[kx] \cos[\omega t] \ ,$$

calculate the magnetic field $\vec{B}$ by integrating the equation obtained from the Faraday law.

3. (10pts) This problem is about harmonic plane-wave solutions to the Maxwell equations. Consider a plane-wave propagating in the direction of a unit vector $\hat{n} = \alpha\hat{i} + \alpha\hat{k}$, where $\alpha = 1/\sqrt{2}$. The plane of polarization of the electric field is the plane $x - z$. The irradiance of this plane wave is $I = 10^6 \mathrm{W/m^2}$, and the wavelength is $\lambda = 400\mathrm{nm}$. Relative permittivity of the medium is $\epsilon_r = 2$,

A) (1pt) What is the polarization vector $\hat{e}$ (i.e. direction of oscillation) of the electric field?

B)(2pt) Determine the electric field amplitude E_0.

C) (2pt) What is the magnetic field amplitude $\vec{B}_0$ in this wave? What direction it has? Use either ϵ_r or refractive index in your answer.

C) (3pt) Give the expression for the energy density of an electromagnetic field, and show that the magnetic and electric parts of the energy density are equal for this plane wave.

D) (1pt) Specify the time-averaged Poynting vector $\langle \vec{S} \rangle_T$.

E) (1pt) Specify the Poynting vector $\vec{S}(x, y, z, t)$ for this plane wave. You should not need to calculate electric and magnetic fields to answer this question.

4. (10pts) Gaussian beam has power $P = 10\text{W}$, and the maximal intensity $I_0 = 5 \times 10^{10}\text{W/m}^2$. It propagates along the z-axis, and the point of the maximal intensity is at the origin of the Cartesian coordinate system. The wavelength is $\lambda = 1.3\mu\text{m}$.

A) (1pt) Calculate the waist w_0 of this beam.

B) (2pt) Calculate the Rayleigh range and explain its physical meaning.

C) (2pt) Calculate the beam spot size at the distance of ten times Rayleigh range.

D) (1pt) Make a sketch of $I(x = 0, y = 0, z)$, i.e. the on-axis intensity versus propagation distance, find out the points at which the intensity equals one half of the maximal, and mark them in your sketch.

E) (2pt) Give the energy of a single photon, and calculate the total photon flux (i.e. number of photons per second) in this beam.

F) (2pt) Calculate the force this beam exerts on a perfectly absorbing object. What is the force on a perfect mirror at normal incidence?