OPTI 210, Spring 2020 **Problem Set 5** Prof. M. Kolesik

Due: Beginning of class, Monday March 16 (15 points)

1. This problem is intended to take you through an example of how the optical dispersion formula obtained in class is often used in combination with experimental data to produce a useful expression for the refractive-index of a dielectric medium. We start from the expression for the frequency dependent refractive-index $n(\omega)$

$$n^2(\omega) = 1 + \left(\frac{Nq_e^2}{m_e\epsilon_0}\right)\frac{1}{(\omega_0^2 - \omega^2)}. \tag{1}$$

This expression is valid for optical frequencies ω far below the material resonant frequency $\omega_0 >> \omega$.

(a) By introducing the optical wavelength $\lambda = 2\pi c/\omega$ and the medium resonant wavelength $\lambda_0 = 2\pi c/\omega_0$ show that Eq. (1) can be rewritten as

$$(n^2(\lambda) - 1)^{-1} = -C\lambda^{-2} + C\lambda_0^{-2}, \tag{2}$$

where the refractive-index is now viewed as a function of the (free-space) wavelength λ, and $\frac{1}{C} = \left(\frac{Nq_e^2}{4\pi^2 c^2 m_e\epsilon_0}\right)$. Equation (2) has the form of a straight line $y = mx + b$, with the identifications $y = (n^2 - 1)^{-1}, x = \lambda^{-2}$, slope $m = -C$, and the intercept $y = b = C\lambda_0^{-2}$ at $x = 0$. What is often done in practice is that $n(\lambda)$ is measured experimentally for a variety of values of λ, $y = (n^2(\lambda) - 1)^{-1}$ is then plotted versus $x = \lambda^{-2}$, and the slope $m = -C$ is determined from the plot. (2)

(b) Here is a table of values for the refractive-index of SCHOTT-FK (Fluorite crown)

λ (nm)	$n(\lambda)$
660.0	1.4852
500.0	1.4914
400.0	1.4994

Write a Matlab code to plot $y = (n^2(\lambda) - 1)^{-1}$ versus $x = \lambda^{-2}$ for the data above, and attach a copy of your code with your solution. You should find a nearly linear relation between y and x. Using this plot extract the slope $m = -C$ from the data, and by extrapolating your plot back to $x = 0$ find the intercept $b = C\lambda_0^{-2}$. (Reasonable estimates of these parameters from your plots will be accepted. For more precision you may try the Matlab fitting routine POLYFIT, but this is not essential.) (4)

(c) Using your numerical values of the slope $m = -C$ and intercept $b = C\lambda_0^{-2}$ determined from part (b) calculate the value of the resonant wavelength λ_0 in nm. (You will find $\lambda_0 \simeq 90$ nm, which is in the UV) (2)

(d) Write a Matlab code to plot $n(\lambda)$ versus wavelength for this glass over the visible range $\lambda = 400 - 2000$ nm, and attach a copy of your code with your solutions. (3)

(e) Go to http://refractiveindex.info, look up SCHOTT-FK, and find out what is the refractive index at = 1000 nm. Use the fit you obtained to evaluate the refractive index for the same wavelength. Give a possible reason for the difference. Hint: You may want to look at the graph of refractive index shown on the website. (2pts)

(f) Explain whether the glass exhibits normal or anomalous dispersion in the visible region. (2)