OPTI 210, Spring 2020 **Problem Set 3** Prof. M. Kolesik

Due: Beginning of class, Wednesday Feb 12 (15 points)

1. This question deals with the three-dimensional (3D) harmonic wave solution

$$\psi(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t+\epsilon)},$$

where A is the amplitude of the wave of propagation vector $\vec{k}$, angular temporal frequency ω, and initial phase ϵ.

(a - 3pts) By going through the explicit calculation in Cartesian coordinates demonstrate that the operator relations (or equivalences) $\nabla \equiv +i\vec{k}$ and $\frac{\partial}{\partial t} \equiv -i\omega$ are obeyed when applied to the above plane-wave solution.

(b - 2pts) Show using the operator relations from part (a) that

$$\nabla^2\psi = -k^2\psi, \qquad \frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi.$$

(c - 2pts) Using the results from part (b) show that demanding that the complex plane-wave solution obeys the 3D wave equation requires that the *dispersion relation* $\omega = ck$ is satisfied, which relates the wave (phase) velocity c, propagation number, and angular temporal frequency.

(d - 2pt) What propagation vector would correspond to a wave propagating at 45 degrees with respect to x and y axis and perpendicular to z axis? Calculate the propagation wave vector and the angular frequency for the speed of light, and the wavelength of 800 nm.

2. Here we consider an outgoing spherical harmonic wave with its source at the origin

$$\psi(\vec{r},t) = \frac{\mathcal{A}}{r}\cos(kr-\omega t),$$

with k the propagation number, ω the angular temporal frequency, and $\mathcal{A}$ and the source strength. Set $k = 2\pi$ and $\mathcal{A} = 1$ for your numerical calculations.

(a - 3pts) Write a Matlab code to plot the field profile $\psi(x,y,z,t=0)$ along the z-axis over the range $z = [2,10]$ for $x = y = 0$, and $t = 0$. This plot should provide a profile of the wave along an axis that passes through the origin, and you should see the $1/r$ variation of the field away from the origin in addition to the expected spatial oscillations of the wave. Please submit a listing of your code along with your numerical plot.

(b - 3pts) Next write a Matlab code to plot the field profile $\psi(x,y,z,t=0)$ along the x-axis over the range $x = [-10,10]$ for $y = 0$, $z = 5$, and $t = 0$. This plot shows the variation of the field profile along an axis which does not pass through the origin. Note the uneven spacing of the fringes close to the origin, but the $1/r$ variation is evident away from the origin. Please submit a listing of your code along with your numerical plot.