OPTI 210, Spring 2020 **Problem Set 2** Prof. M. Kolesik

Due: Beginning of class, Wed. Fe. 5 (15 points)

1. Consider a pulsed (i.e. finite-duration) wave of the form $\psi(x,t) = \exp[-(x-15+vt)^2]\cos(10(x+vt))$. This function may approximate, e.g., an ultrafast optical pulse with a gaussian envelope.

(A - 4pts) Write a Matlab (or other language) code to plot this solution over the range $x = [-20, 25]$ for the parameters $v = 1$, and $t = 0, 15, 30$. Use different plotting colors for each time so that you can see how the solution evolves with time, and note the value of x at which the function is peaked for each time t. Please submit a listing of your code along with your numerical plot. Make sure that your plot has a decent resolution (number of plot points) in order to show details.

(B - 1pts) Identify which of the two functions that constitute this pulsed waveform (i.e. exp, and cos) should be identified as the (faster changing) carrier wave and which plays the role of the (slower changing) envelope.

2. This question deals with the harmonic wave solution

$$\psi(x,t) = A\sin(kx \pm \omega t + \pi/2),$$

where A is the amplitude of the wave form of propagation number k, and angular temporal frequency ω.

(A - 2pts) Show by direct evaluation that for the harmonic wave solution above

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi, \qquad \frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi.$$

(B - 1pt) Using the results from part (A) show that demanding that the harmonic wave solution obeys the 1D wave equation requires that the *dispersion relation* $\omega = vk$ is satisfied, which relates the wave (phase) velocity, propagation number, and angular temporal frequency.

(C - 1pt) For which sign in front of ω does this wave propagate to the left?

(D - 1pt) For the wavelength range $\lambda = 300-700$ nm, and $v = 3\times10^8$ m/s, what is the corresponding ranges of spatial frequencies in m^{-1}, angular temporal frequencies in rads^{-1}, and temporal frequencies in s^{-1}?

3. Here we consider the interference between two harmonic waves at $t = 0$

$$\psi(x,t=0) = \sin(k_1 x) + \sin(k_2 x). \tag{1}$$

(A - 1pt) Using Matlab produce a plot of $\psi(x,t=0)$ according to Eq. (1) on the interval $x = [0, 30]$ for $k_1 = 2\pi, k_2 = 1.8\pi$.

(B - 2pts) By using the rules for the sum of trigonometric functions show that

$$\psi(x,t=0) = 2\sin(K_+x)\cos(K_-x), \tag{2}$$

with $K_\pm = (k_1 \pm k_2)/2 > 0$. The interference of the two waves can therefore be expressed as a product of cosine functions with different propagation numbers $K_\pm$.

(C - 2pts) For $k_1 \approx k_2$ the *average* propagation number $K_+ = (k_1+k_2)/2 \approx k_1$ controls the fast carrier oscillations that arise in your plot from part (A), whereas the *"spread"* $K_- = (k_1-k_2)/2 << K_+$ describes the slower spatial envelope that modulates the carrier oscillation. The spatial periods $L_\pm$ of the carrier and envelope fringes follow from the relations $K_\pm L_\pm = 2\pi$, which requires the harmonic waves to repeat in phase after a period. Demonstrate that the spatial periods $L_+ \approx 2\pi/k_1$ and $L_- \approx 4\pi/(k_1-k_2)$ indeed appear in your numerical results from part (A).