OPTI 210, Spring 2020 **Problem Set 1** Prof. M. Kolesik

**Due: Beginning of class, Wednesday Jan. 30** (20 points)

**1.** This problem deals with aspects of vector analysis. Three vectors are specified as follows: $\vec{A} = 2\hat{i} + 5\hat{j}$, $\vec{B} = \hat{i} - 4\hat{j} + 2\hat{k}$, and $\vec{C} = C_x\hat{i} + C_y\hat{j}$.

(a - 1pt) Given that the vectors $\vec{A}$ and $\vec{C}$ are orthogonal and that $\vec{C}$ is a unit vector, calculate values for $C_x$ and $C_y$.

(b - 1pt) Without resort to calculation provide an example of a unit vector that is automatically orthogonal to both $\vec{A}$ and $\vec{C}$.

(c - 1pt) Calculate the scalar product of $\vec{A}$ and $\vec{B}$.

(d - 1pt) Calculate the cross product of $\vec{A}$ and $\vec{B}$.

(e - 1pt) Calculate angle $\theta$ between $\vec{A}$ and $\vec{B}$.

**2.** Scalar function $h$ is given like so:

$$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12),$$

(a - 2pt) Calculate the gradient of $h$

(b - 2pt) Calculate the Laplacian of $h$

**3.** Vector function $\vec{F}$ is given like so:

$$\vec{F}(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}(x^2+y^2)^k\right)\hat{i} - \left(\frac{x}{\sqrt{x^2+y^2}}(x^2+y^2)^k\right)\hat{j},$$

(a - 2pt) Calculate divergence of $\vec{F}$

(b - 2pt) Calculate curl of $\vec{F}$ (Note: think before calculating, you may want to simplify first.)

(c - 2pt) Decide if $\vec{F}$ has a potential, i.e. if there exists a function $U$ such that $\vec{F} = \nabla U$.

**4.** This problem deals with the use of complex numbers and the Euler formula

(a - 1pt) Calculate the magnitude and phase of the complex number $z = 9 - 12i$.

(b - 1pt) Show that the magnitude of a complex number in the form

$$z = \frac{a+ib}{a-ib}$$

is always equal to one for for real $a$ and $b$. We shall encounter such form for the reflection coefficient in the total internal reflection...

(c - 1pt) By expressing the complex function $Ae^{-i\omega t}$ ($\omega$ is real) in terms of its real and imaginary parts, with $A = A_r + iA_i$ complex, show that the magnitude of the complex function is given by $|A|$.

(d - 2pts) Using complex notation, prove that $\sin(3\theta) = [3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)]$, (Hint: You will want to make use of this: $e^{3i\theta} = (e^{i\theta})^3$).